



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

### Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

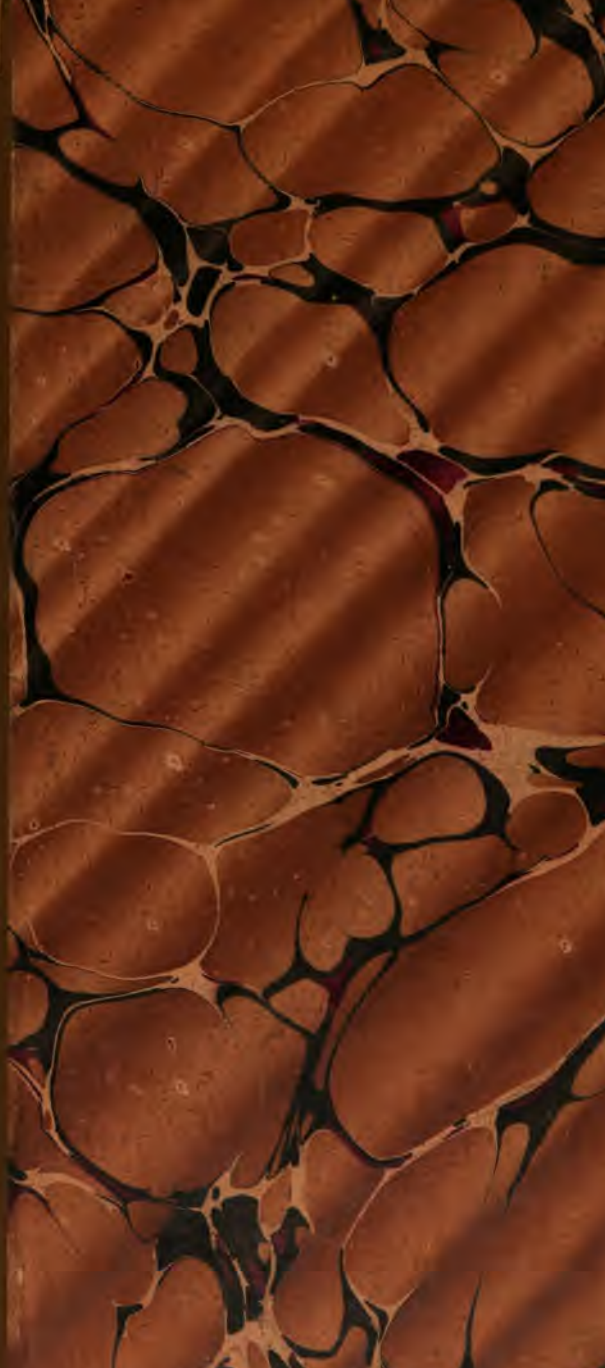
- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

### About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

Wentworth · Answers to Exercises  
in Algebra · 1885

128  
85.874




Edvc T128.85.874



HARVARD  
COLLEGE  
LIBRARY

THE GIFT OF

*Miss Ellen Lang Wentworth  
of Exeter, New Hampshire*



3 2044 097 011 480



# ANSWERS

TO THE

PROBLEMS IN WENTWORTH AND HILL'S

EXERCISES IN ALGEBRA.

---

Part I.: Exercise Manual.

Part II.: Examination Manual.

---

BOSTON :

PUBLISHED BY GINN & COMPANY.

1885.

**WENTWORTH'S**  
**SERIES OF MATHEMATICS.**

---

Primary School Arithmetic.  
Grammar School Arithmetic.  
Practical Arithmetic.  
Practical Arithmetic (*Abridged Edition*).  
Exercises in Arithmetic.  
Shorter Course in Algebra.  
Elements of Algebra.  
Complete Algebra.  
University Algebra.  
Exercises in Algebra.  
Plane Geometry.  
Plane and Solid Geometry.  
Exercises in Geometry.  
Pl. and Sol. Geom. and Pl. Trigonometry.  
Plane Trigonometry and Tables.  
Pl. and Sph. Trig., Surveying, and Tables.  
Trigonometry, Surveying, and Navigation.  
Log. and Trig. Tables (*Seven*).  
Log. and Trig. Tables (*Complete Edition*).  

---

**Special Circular and Terms on application.**





Edvc T 128.85.874



HARVARD  
COLLEGE  
LIBRARY

THE GIFT OF

*Miss Ellen Lang Wentworth  
of Exeter, New Hampshire*



3 2044 097 011 480

15.  $-1700b^{3n-m+4}d^{3p} + 1350b^{n+3}d^{3p+q+2} - 25b^{n-2m+5}d^{3p+3q-1}$ .
16.  $1170d^{1+3p} - 1620d^3 + 1800d^{2+5p}$ .
17.  $600x - 1050x^{m-5n+4} + 625x^{2m}$ .
18.  $42y^p - 39y^{3-4m} + 3y^{4p+1}$ .      21.  $4x^2 - 11xy - 45y^2$ .
19.  $3a^3 - 7ab + 4b^2$ .      22.  $6a^2 - 7ab + 2b^2$ .
20.  $12b^2 + bc - 20c^2$ .      23.  $5a^4 + 7a^3b - 8a^2b^2 - 4ab^3$ .
24.  $16x^4 - 32x^3y + 16x^2y^2 - 8xy^3 + 3y^4$ .
25.  $3a^4 - 26a^3b + 37a^2b^2 - 14ab^3$ .
26.  $10a^5 - 9a^4x + 9a^3x^2 - 3a^2x^3 + ax^4$ .
27.  $15x^4 - 23x^3 + 27x^2 + 9x - 28$ .
28.  $x^4 + 4x^3 - 19x^2 + 64x - 35$ .
29.  $6a^5b - 7a^4b^2 - 11a^3b^3 + 9a^2b^4 - 5ab^5$ .
30.  $x^5 - 10ax^4 + 40a^2x^3 - 80a^3x^2 + 80a^4x - 32a^5$ .
31.  $x^5 - 5x^4 + 3x^3 + 6x^2 - 7x + 2$ .
32.  $x^7 - 9x^5y^2 + 7x^4y^3 + 13x^3y^4 - 19x^2y^5 + 8xy^6 - y^7$ .
33.  $x^4 - 3ax^3 - 9a^2x^2 - 19a^3x - 6a^4$ .
34.  $4x^5 - 16x^4 - 16x^3 + 12x^2 + 32x^3 + 24x^2 + 8x + 1$ .
35.  $x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$ .
36.  $a^3 - 6a^2b + 9a^2c + 12ab^2 - 36abc + 27ac^2 - 8b^3$   
 $+ 36b^2c - 54bc^2 + 27c^3$ .
37.  $6x^6 + 3x^5y - 15x^4y^2 - 12x^3y^3 + 51x^2y^4 - 45xy^5 + 12y^6$ .
38.  $-6a^5x + 22a^4x^2 - 34a^3x^3 + 54a^2x^4 - 68ax^5 + 32x^6$ .
39.  $2by^6 - 11b^2y^7 + 28b^3y^8 - 39b^4y^5 + 44b^5y^4 - 30b^6y^3$ .
40.  $6c^5 - 50c^4z + 58c^3z^2 - 40c^2z^3 + 38cz^4 + 132z^5$ .
41.  $a^6 - 10a^5b + 29a^4b^2 - 24a^3b^3 - 14a^2b^4 + 22ab^5 - 4b^6$ .
42.  $x^6 - 2x^5y - x^4y^2 + 4x^3y^3 - x^2y^4 - 2xy^5 + y^6$ .
43.  $6a^6b^3 - 27a^5b^4 + 44a^4b^5 - 30a^3b^6 + 3a^2b^7 + 2ab^8$ .
44.  $3x^7 - 11x^6 + 7x^5 + 11x^4 - 2x^3 + x^2 - 28x + 15$ .
45.  $35a^2 + 11a^3 - 15a^4 + 18a^5 - 68a^6 + 28a^7$ .

46.  $10p^5 + 16p^4q - 25p^3q^2 - 3p^2q^3 + 5pq^4 + q^5$ .
47.  $2a^7 - 5a^6b + 9a^5b^2 - 9a^4b^3 + 7a^3b^4 - 5a^2b^5 + 2ab^6 - b^7$ .
48.  $45c^{14} - 2c^{12} + c^{10} - 6c^8 - 131c^6 + 150c^4 - 52c^2$ .
49.  $a^8 - 15a^5b + 72a^4b^2 - 95a^3b^3 - 72a^2b^4 - 15ab^5 - b^6$ .
50.  $x^{12} - 12x^{11} + 46x^{10} - 36x^9 - 111x^8 + 72x^7 + 184x^6$   
 $+ 96x^5 + 16x^4$ .
51.  $14d^8 - d^7e - 7d^6e^2 - 485d^5e^3 + 50d^4e^4 - 436d^3e^5$   
 $+ 630d^2e^6 + 548de^7 - 16e^8$ .
52.  $3a^5 - 7a^4x + 8a^3x^2 - 8a^2x^3 + 3\frac{1}{8}ax^4 - x^5$ .
53.  $50c^7 - 82c^6 - 61\frac{1}{10}c^5 + 5990\frac{7}{10}c^4 + 1761\frac{1}{10}c^3 - 155\frac{3}{4}c^2$   
 $- 59\frac{1}{5}c + 4$ .
54.  $8d^8 - 33cd^7 - 32\frac{4}{5}c^2d^6 - 70\frac{1}{5}c^3d^5 - 56\frac{1}{2}c^4d^4 + 2\frac{5}{12}c^5d^3$   
 $+ 8\frac{5}{8}c^6d^2 - \frac{1}{4}c^7d$ .
55.  $-6p^8 + 201p^7q - 116p^6q^2 - 649p^5q^3 + 448p^4q^4$   
 $+ 187\frac{7}{8}p^3q^5 - 328p^2q^6 + 179\frac{1}{2}pq^7 - 5\frac{1}{2}q^8$ .
56.  $168c^9 - 211\frac{1}{8}c^8d + 34c^7d^2 + 82c^6d^3 - 139\frac{3}{8}c^5d^4$   
 $+ 72\frac{1}{8}c^4d^5 + \frac{2}{3}c^3d^6 - \frac{1}{8}c^2d^7$ .
57.  $\frac{c^8}{27} - \frac{c^5}{12} - \frac{5c^4}{48} + \frac{15c^3}{64} + \frac{5c^2}{32} - \frac{3c}{16} - \frac{1}{8}$ .
58.  $\frac{x^2}{4} - \frac{x^3}{3} + \frac{13x^4}{36} - \frac{x^5}{6} + \frac{x^6}{16}$ .
59.  $0.2m^6 - 0.46m^5n + 0.32m^4n^2 - 0.08m^3n^3 - 2m^2n^4$   
 $+ 0.2mn^5 + 1.12n^6$ .
60.  $14x^5 - 6x^4y - 7x^3 + 2x^3y^2 + 10x^2y - 4xy^2 + y^3$ .
61.  $14x^9 + 21x^8 - 6x^7 + 38x^6 + 7x^5 + 4x^4 - 30x^3 + 26x^2$   
 $- 36x - 20$ .
62.  $x^7 - 2\frac{1}{8}x^6y + 7x^5y^2 - 10\frac{1}{8}x^4y^3 + 12\frac{3}{8}x^3y^4 - 8\frac{3}{8}x^2y^5$   
 $+ 6xy^6 - 2y^7$ .
63.  $9b^8 - 30b^5d + 40b^4d^2 - 46b^3d^3 + 44b^2d^4 - 26bd^5 + 3d^6$ .

64.  $8a^9 - 36a^8b - 30a^7b^2 + 225a^6b^3 + 105a^5b^4 - 441a^4b^5 - 343a^3b^6.$
65.  $27p^7 - 21p^6x + 18p^5x^2 - 74p^4x^3 - 29p^3x^4 + 23p^2x^5 + 72px^6.$
66.  $8b^6 + 10b^5 + 4b^4 - 21b^3 - 6b^2 + 5b.$
67.  $3 + 7a - 8a^2 + 19a^3 - 10a^4 + 4a^5.$
68.  $a^2b + a^2c + abc - ab^2 + ac^2 - b^2c + bc^2.$
69.  $1 - 2x + 4xy - 2xy^2 + x^2 - 2x^2y + x^2y^2 - 2y + y^2.$
70.  $b^2cx^2z - bc^2x^2y + 3abcxyz - ac^2xy^2 - ab^2xz^2 + a^2cy^2z - a^2byz^2.$
71.  $x^4 - (a - b)x^3 - (ab - b + c)x^2 + (ac + b^2)x - bc.$
72.  $x^4 - (a - p)x^3 - (b^2 + ap + q^2)x^2 + (aq^2 - b^2p)x + b^2q^2.$
73.  $x^4 - (3a - 5b)x^3 - (6a^2 + 15ab - 8b^2)x^2 - 6ab(5a + 4b)x - 48a^2b^2.$
74.  $x^5 - (a + m)x^4 + (am + b + n)x^3 - (an + bm + c)x^2 + (bn + cm)x - cn.$
75.  $x^5 - (a - m)x^4 + (b + n - am)x^3 - (an - bm + c)x^2 + (bn - cm)x - cn.$
76.  $cx^5 - (b + c)x^4 + (a + b - c)x^3 - (a - b + 1)x^2 + (1 - a)x + 1.$
77.  $adz^4 + (ae + bd)z^3 + (af + be + cd)z^2 + (bf + ce)z + cf.$
78.  $mrz^5 - (nr - ms)z^4 + (mp - ns - pr)z^3 - (mu + np + ps - qr)z^2 + (nu - p^2 + qs)z^2 + p(q + u)z - qu.$
79.  $3ma^5 + (9m - 4n)a^4 - (3m + 3n - p)a^3 + (4n - 3p)a^2 - 4pa.$
80.  $8amx^5 + 2(5an - 6bm)x^4 - (4ap + 15bn - 16dm)x^3 - 2(av - 3bp - 10dn)x^2 + (3bv - 8dp)x - 4dv.$
81.  $a^m + 3a^{m+1} - a^{m+2} - 3a^{m+3} - a - 1.$
82.  $a^{n+1} - 5a^n + 9a^{n-1} - 4a^{n-2} - a^{n-3}.$

83.  $a^{n+1} - 2a^n b + a^{n-1} b^2 - a^{n-2} b^3 + a^{n-3} b^{n+1}$ .
84.  $x^n + x^{n-5} y^5$ .
85.  $2a^{p+3} - 7a^{p+2} + 12a^{p+1} - 19a^p + 20a^{p-1} - 11a^{p-2} + 5a^{p-3}$ .
86.  $a^{2n+3} - 2a^{2n+2} - 2a^{2n+1} + a^{n+3} + 2a^{n+2} - a^{n+1} - 2a^n + a^{n-1}$ .
87.  $6x^{2n+2} - 23x^{2n+1} + 18x^{2n} - x^{2n-1} - 3x^{2n-2} + 4x^{2n-3} - x^{2n-4}$ .
88.  $m^{p+r} - 11m^{p+r-2}n^2 - 13m^{p+r-3}n^3 + 3m^{p+r-5}n^5$ .
89.  $e^{2m} - 2e^{2m-1}d + 3e^{2m-2}d^2 - 4e^{2m-3}d^3 + 3e^{2m-4}d^4 - 2e^{2m-5}d^5 + e^{2m-6}d^6$ .
90.  $4x^{2r+7}y^{1-2r} - 9x^{2r+6}y^{1-r} + 22x^{2r+4}y - x^{2r+3}y^{1+r} + 16x^{2r+1}y^{1+2r}$ .
91.  $2a^3b - 18a^4b^6 + 6a^3b^7 - 18a^3b^8 + 4ab^9$ .
92.  $2u^{r+1} - 6u^{r+2} + 2u^{r+3} - 4u^{r+4} + 3u^{2r+1} - 9u^{2r+2} + 3u^{2r+3} - 6u^{2r+4} - 4u^{2r+1} + 12u^{2r+2} - 4u^{2r+3} + 8u^{2r+4}$ .
93.  $-8x^{2p+4}y^{2r-1} - 10x^{2p+3}y^{2r} + 19x^{2p+2}y^{2r+1} + 16x^{2p+1}y^{2r+2} - 5x^{2p}y^{2r+3}$ .
94.  $a^{2p+2} - 4a^{2p} + 12a^{2p-1} - 9a^{2p-2}$ .
95.  $9y^{m+p-3} - 34y^{m+p-1} + 29y^{m+p+1} - y^{m+p+3}$ .

## Ex. 5. Page 13.

- |                                   |                             |
|-----------------------------------|-----------------------------|
| 1. $5a^5 + 5$ .                   | 12. $3a^2x - 4ax^2 + x^3$ . |
| 2. $7x^2 + 8x + 1$ .              | 13. $y^2 - 5y + 6$ .        |
| 3. $x^2 + bz^2 + b^2z + b^3$ .    | 14. $2y^3 - 4y^2 + y - 1$ . |
| 4. $2a^2 + 1$ .                   | 15. $2x - 7y$ .             |
| 5. $6a^3 - a - 1$ .               | 16. $2a^2 + 5a - 7$ .       |
| 6. $x^3 + 3x^2 - x - 3$ .         | 17. $3a^3 - 2ab - b^2$ .    |
| 7. $b^3 + b^2 - 5b + 3$ .         | 18. $3x^2 - 2ax + 5a^2$ .   |
| 8. $12m^3 + 20m^2n + 8mn^2$ .     | 19. $a^4 + a^2x^2 + x^4$ .  |
| 9. $d^4 - 2d^3 + 3d^2 - 2d + 1$ . | 20. $5x + 2$ .              |
| 10. $2m^2 + 1$ .                  | 21. $3x^2 + 4xy + y^2$ .    |
| 11. $p^2 + p + 1$ .               | 22. $x^2 - a^2$ .           |

23.  $x^2 + 5xy + 3y^2$ .  
 24.  $2x^2 + 5xy - y^2$ .  
 25.  $x^2 - 7x + 5$ .  
 26.  $x^2 - 3x + 1$ .  
 27.  $3x^2 + 2xy - y^2$ .  
 28.  $x^2 + 5x + 6$ .  
 29.  $5x^4 - 2x^2y + 4x^2y^2$ .  
 30.  $7a^2 - 3ab + 2b^2$ .  
 31.  $\frac{4}{3}b^2 - \frac{2}{3}bd + \frac{1}{4}d^2$ .  
 32.  $\frac{1}{3}d^3 - \frac{2}{3}d^2 + \frac{1}{2}d$ .  
 33.  $\frac{3}{8}a^3 - \frac{2}{3}a^2b + \frac{1}{2}ab^2$ .  
 34.  $\frac{1}{4}m^2 + \frac{3}{4}mn - \frac{2}{3}n^2$ .  
 35.  $\frac{2}{3}x^2 + \frac{3}{4}xy - \frac{1}{2}y^2$ .  
 36.  $\frac{2}{3}a^3 - \frac{1}{2}a^2b + \frac{5}{6}ab^2 - \frac{1}{3}b^3$ .  
 37.  $x^2 - \frac{3}{2}x^2 - 2x + \frac{1}{2}$ .  
 38.  $\frac{1}{2}a^2 + \frac{2}{3}a^2b - \frac{1}{4}ab^2 + \frac{3}{4}b^3$ .  
 39.  $1\frac{1}{2}y^4 - \frac{3}{4}y^3z - \frac{3}{8}y^2z^2 + \frac{1}{2}yz^3 + \frac{3}{8}z^4$ .  
 40.  $2\frac{1}{2}z^5 + 3\frac{1}{4}z^4 - 6\frac{1}{2}z^3 + \frac{3}{8}z^2 - \frac{1}{8}z + \frac{7}{8}$ .  
 41.  $-\frac{5}{2}a^3 - \frac{3}{4}a^2b + \frac{1}{2}ab^2 + 6b^3$ .  
 42.  $-(4a + x)$ .  
 43.  $8x - 5$ .  
 44.  $x^4 - 3x^3y - 5x^2y^2 + 7xy^3 - 2y^4$ .  
 45.  $4x^3 - 5x^2y - 8xy^2 + 2y^3$ .  
 46.  $a^4 - 3a^3b + 4a^2b^2 - 2ab^3 + 4b^4$ .  
 47.  $3x - 8x^2y + 7x^3y^2 - 5x^4y^3 + 7x^5y^4$ .  
 48.  $5y + 7y^2z - 9y^3z^2 + y^4z^3$ .  
 49.  $\frac{5}{8}a^3 - \frac{2}{3}a^2x + \frac{1}{4}ax^2 - \frac{1}{8}x^3$ .  
 50.  $a^4 - 7a^2b^2 + 6ab^3 - b^4$ .  
 51.  $2a^2 - 6a + 2$ .  
 52.  $7x^2 + 3xy - 9y^2$ .  
 53.  $2x^2y + 2x^2y^2 - xy^3$ .  
 54.  $2x^2 + xy - 3y^2$ .  
 55.  $x^3 + 3x^2 + x - 2$ .  
 56.  $x^5y^5 + 2x^4y^4 + 4x^3y^3 + 8x^2y^2 + 16xy + 32$ .  
 57.  $y^9 + y^7 + y^6 + y^5 + y^4 + y^3 + y^2 + 1$ .  
 58.  $a - b$ .  
 59.  $3x^2 - 3xy + 4y^2$ .  
 60.  $2a^2 + 3ab - b^2$ .  
 61.  $(a + b)^3$ .  
 62.  $a^2 + 2ab + b^2 + c$ .  
 63.  $2x^2 + 3xy + y^2 - 2$ .  
 64.  $a - 2a^2 + 3a^3$ .  
 65.  $a^m + 2a^{m+1} - 4a^{m+2} + 5a^{m+3}$ .  
 66.  $3d^x - 4d^{x-1} + 5d^{x-2} - d^{x-3}$ .  
 67.  $4h^4 - 2h^3 - 3h^2$ .

68.  $2x^{m-1} + 3x^{m-2} - 4x^{m-3}$ . 72.  $x^{(m-1)n} - y^{(n-1)m}$ .
69.  $3b^x - 4b^{x-1} + 3b^{x-2} - b^{x-3}$ . 73.  $x^{2n} - 2x^n y^n + y^{2n}$ .
70.  $3x^{4n+1} - 4x^{3n} + 2x^{2n-1} - x^{n-2}$ . 74.  $a^n b^n - 7a^{n-1} b^{2n}$ .
71.  $a^m + 3a^{m-1} b^n - 6a^{m-2} b^{2n}$ . 75.  $1 - x + x^2 - x^3 + x^4 - \dots$
76.  $1 + x + x^2 + x^3 + x^4 + \dots$
77.  $1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \frac{1}{x^4} - \dots$
78.  $\frac{1}{x} + \frac{1}{x^3} + \frac{1}{x^5} + \frac{1}{x^7} + \frac{1}{x^9} + \dots$
79.  $a - ax + ax^2 - ax^3 + ax^4 - \dots$
80.  $m + my + my^2 + my^3 + my^4 + \dots$
81.  $1 + 5x + 15x^2 + 45x^3 + 135x^4 + \dots$
82.  $1 - 4a + 8a^2 - 16a^3 + 32a^4 - \dots$
83.  $1 + \frac{2y}{x} + \frac{2y^2}{x^2} + \frac{2y^3}{x^3} + \frac{2y^4}{x^4} + \dots$
84.  $1 - (a+b)x + (a+b)bx^2 - (a+b)b^2x^3 + (a+b)b^3x^4 - \dots$
85.  $\frac{a}{b} + \frac{a-b}{b^2y} + \frac{a-b}{b^3y^2} + \frac{a-b}{b^4y^3} + \frac{a-b}{b^5y^4} + \dots$   
 or  $\frac{a}{b} + \frac{a}{b^2y} + \frac{a}{b^3y^2} + \frac{a}{b^4y^3} + \dots$
86.  $1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots$
87.  $\frac{1}{x} + \frac{2}{x^2} + \frac{4}{x^3} + \frac{8}{x^4} + \frac{16}{x^5} + \dots$
88.  $\frac{1}{3} - \frac{x}{9} + \frac{x^2}{27} - \frac{x^3}{81} + \frac{x^4}{243} - \dots$
89.  $-\frac{1}{2} - \frac{x}{4} - \frac{x^2}{8} - \frac{x^3}{16} - \frac{x^4}{32} - \dots$
90.  $1 + 3x + 5x^2 + 7x^3 + 9x^4 + \dots$



---

<p>91. <math>1 + \frac{x}{1} = 2</math></p> <p><math>\frac{x^2}{1 \times 2} = 0.5</math></p> <p><math>\frac{x^3}{1 \times 2 \times 3} = 0.1666666</math></p> <p><math>\frac{x^4}{1 \times 2 \times 3 \times 4} = 0.0416666</math></p> <p><math>\frac{x^5}{1 \times 2 \dots 5} = 0.0083333</math></p> <p><math>\frac{x^6}{1 \times 2 \dots 6} = 0.0013888</math></p>	<p><math>\frac{x^7}{1 \times 2 \dots 7} = 0.0001984</math></p> <p><math>\frac{x^8}{1 \times 2 \dots 8} = 0.0000248</math></p> <p><math>\frac{x^9}{1 \times 2 \dots 9} = 0.0000027</math></p> <p><math>\frac{x^{10}}{1 \times 2 \dots 10} = 0.0000003</math></p> <p><math>\frac{x^{11}}{1 \times 2 \dots 11} = 0.0000000</math></p>
---	--

$$\begin{array}{r} 2.7180555 \\ 0.0002262 \\ \hline = 2.7182817 \text{ Ans.} \end{array}$$

<p>92. <math>1 + \frac{1}{5} = 1.2</math></p> <p><math>\frac{1}{9} = 0.1111111</math></p> <p><math>\frac{1}{13} = 0.0769230</math></p> <p><math>\frac{1}{17} = 0.0588235</math></p> <p><math>\frac{1}{21} = 0.0476190</math></p> <p><math>\frac{1}{25} = 0.04</math></p> <p><math>\frac{1}{29} = 0.0344827</math></p>	<p><math>\frac{1}{3} = 0.333333</math></p> <p><math>\frac{1}{7} = 0.142857</math></p> <p><math>\frac{1}{11} = 0.090909</math></p> <p><math>\frac{1}{15} = 0.066666</math></p> <p><math>\frac{1}{19} = 0.052631</math></p> <p><math>\frac{1}{23} = 0.043478</math></p>
---	---

$$\begin{array}{r} 1.5689593 \\ - 0.729874 \\ \hline 0.849085 \times 4 = 3.396340. \text{ Ans.} \end{array}$$

---

93.  $\frac{a}{1} = 0.2$   $\frac{a^3}{3} = 0.002666666$   
 $\frac{a^5}{5} = 0.000064$   $\frac{a^7}{7} = 0.000001828$   
 $\frac{a^9}{9} = 0.000000056$   $\frac{a^{11}}{11} = 0.000000002$   
 $\quad\quad\quad 0.200064056$   $\quad\quad\quad 0.002668496$   
 $\quad\quad\quad - 0.002668496$   
 $\quad\quad\quad = 0.19739556 \text{ Ans.}$

94.  $x = 1.5708$   $\frac{x^3}{1 \times 2 \times 3} = 0.6459686291$   
 $\frac{x^5}{1 \times 2 \dots 5} = 0.079693558$   $\frac{x^7}{1 \times 2 \dots 7} = 0.0046818307$   
 $\frac{x^9}{1 \times 2 \dots 9} = 0.000160444$   $\frac{x^{11}}{1 \times 2 \dots 11} = 0.0000035989$   
 $\frac{x^{13}}{1 \times 2 \dots 13} = 0.000000056$   $\quad\quad\quad 0.6506540587$   
 $\quad\quad\quad 1.650654058$   
 $\quad\quad\quad - 0.650654058$   
 $\quad\quad\quad = 1.0 \text{ Ans.}$

95.  $\frac{1}{2n+1} = 0.333333333$   
 $\frac{1}{3(2n+1)^3} = 0.0123456790$   
 $\frac{1}{5(2n+1)^5} = 0.0008230453$   
 $\frac{1}{7(2n+1)^7} = 0.0000653210$   
 $\frac{1}{9(2n+1)^9} = 0.0000056450$   
 $\frac{1}{11(2n+1)^{11}} = 0.0000005131$   
 $\frac{1}{13(2n+1)^{13}} = 0.0000000482$   
 $\frac{1}{15(2n+1)^{15}} = 0.0000000046$   
 $\quad\quad\quad 0.3465735895 \times 2$   
 $\quad\quad\quad = 0.69314718 \text{ Ans.}$

## Ex. 6. Page 17.

1.  $4a - 2c$ .
2.  $4a$ .
3.  $8a - 9b - 8c$ .
4.  $3a - 3x + c$ .
5.  $21 - 11x$ .
6.  $4c$ .
7.  $-8x^2 - 8x$ .
8.  $2bce + 2bef$ .
9.  $8 + 3x$ .
10.  $6x - 5y$ .
11.  $12a - 3x$ .
12.  $(2a^2 - a - 3)b^3 + (7a^2 - 9a - 1)b^2 + (4a^2 - 8a + 9)b - (4a^2 - 12a + 5)$ .
13.  $(a^3 - b^3)m^5 + (a^3 + 2ab + 3b^3)m^4 - (a^2b - ab^2 + 2a + 2b + a^2 + ab + b^2)m^3 + (a + b - 2ab)m^2 + abm$ .
14.  $2(a - b)x^5 - (a^2 - 2ab - b^2)x^4 + 2(a + b - ab^2)x^3 - (2a^2 + 2b^2 - a^2b^2)x^2 + 2ab^2x - (a^3 - b^3)$ .
15.  $x^5 + 2x^5y - (a^2 - 2a + 3)x^4y^2 + (a - 6)x^3y^3 - (7a - 2)x^2y^4 - (a^2 - 4)xy^5$ .
16.  $(b^3 - a^3)m^5 - (a^2 + 3b^2 - 4ab)m^4 + 2(a^2 - 3ab + 2b^2)m^3 - 2(3a^3 - 5ab + 2b^3)m^2 + 4a(a - b)m$ .
17.  $(a^3 - 4b^2 + 2a + 1)x^4 + 4b(4a + 1)x^3 - (7a^2 + 6ab - a + 1)x^2 + (6a^2 + 10ab - 3a - 2b - 4b^3)x - 2b(2a - 1)$ .
18.  $(b^4 - 2b^3 + 1)x^4 - 2(b^3 - 2b^2 - b + 2)x^3 - (2b^3 - 3b^2 + 2b - 2)x^2 + 2(b^3 - 3b + 2)x + (1 - b)^2$ .
19.  $729a^5 - 243(1 - m)a^5 - 54(4 - 5m^2 + m)a^4 + (53 - 51m - 57m^2 + 55m^3)a^3 + 6(4 + m - 9m^2 - m^3 + 5m^4)a^2 - 3(1 - m - 2m^2 + 2m^3 + m^4 - m^5)a - (1 - m^2)^3$ .

NOTE. In this example put  $x$  for  $1 - m$  and  $y$  for  $1 - m^2$ . The multiplication then gives  $729a^5 - 243xa^5 - (243y - 27x^2)a^4 + (54xy - x^5)a^3 + (27y^2 - 3x^2y)a^2 - 3xy^2a - x^5$ . Then,  $x$  and  $y$  are replaced by their values.

20.  $(4a^2 - 9c^2)y^4 - 2acy^3 - (a^2 - 16ac - c^2)y^2 + 2(2a^2 + c^2)y - (4a^2 - c^2)$ .

21.  $x^5 + 2ax^4 - (a^2 - a + 3)x^3 + (a^3 - 4a^2 + 7a - 9)x^2 - (a^3 - 9a^2 + 22a - 10)x + (a^5 - 8a^4 + 24a^3 - 36a^2 + 29a - 7).$
22.  $y^5 + 2by^4 + (a^2 - 2ab + b^2)y^3 + (a^3 - 3a^2b + 3ab^2 - b^3)y^2 + 2(a^3b - 3a^2b^2 + 3ab^3 - b^4)y + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5).$
23.  $2(x^2 + y^2 - z^2).$
24.  $-2(10a^4 - 15a^3 + 7a^2b + 12ab - 12b^2).$
25.  $3a^2b - 2ab^2.$     26. 0.    29. 0.
27.  $-10(a^3 - a^2b - ab^2 + b^3).$     30.  $37b^2 - ab - 2b + 2a - 9a^2.$
28.  $12y^2 - 28xy.$     31.  $5x^4 - 5x^3 - 3x^2 + 3x.$
32.  $-5a^3 + 9a^2c + 3a^2b - 9abc + 2ab^2.$
33.  $10a^4b - 19a^3b^2 + 8a^2b^3 + ab^4 - a^2b + 2ab^2 - b^3.$
34.  $(x+y)^2(2a^2x+x+y)$   
 $= (2a^2+1)x^3 + (4a^2y+3y)x^2 + (2a^2y^2+3y^2)x + y^3.$
35.  $a^2 + ax - 3.$     38.  $a + 2b - c + 3d.$
36.  $a^2 - pa + q.$     39.  $x^2 - (y-1)x + (y^2 + y + 1).$
37.  $z^2 + (a-b)z - a^2.$     40.  $(a-1)x + a^2.$
41.  $(x^2 + x + 1)y - (x + 1).$
42.  $(x^3 + x^2)(a - 2y) + x(y - 2a)y + ay^2.$
43.  $z^2 + (m+n)z - mn.$     45.  $4(a+b)^2 - 6(a+b)x + 8x^2.$
44.  $my^3 + ny^2 + oy + p.$     46.  $4x^3 - 5x^2y - 3xz^2 + 6z^3.$

## Ex. 7. Page 20.

1.  $x^2 + 2xy + y^2.$     6.  $4x^2 - 4ax + a^2.$
2.  $b^2 - 2bc + c^2.$     7.  $27a^2 - 27a^2b + 9ab^2 - b^3.$
3.  $m^3 + 3m^2n + 3mn^2 + n^3.$     8.  $16a^2 + 25b^2 - 40ab.$
4.  $a^3 - 3a^2b + 3ab^2 - b^3.$     9.  $9x^2 + 30xy + 25y^2.$
5.  $m^2 + 4mn + 4n^2.$     10.  $64z^2 - 80z + 25.$

- 
- |   |                                       |
|---|---------------------------------------|
| 11. $9 + 12d + 4d^2$ .                              | 17. $49c^2 - 56cd + 16d^2$ .          |
| 12. $16m^2 - 40mn + 25n^2$ .                        | 18. $64m^3 + 64mn + 16n^2$ .          |
| 13. $1 + 4x^2 + 4x^4$ .                             | 19. $81p^3 - 54pq + 9q^2$ .           |
| 14. $9 + 30m^5 + 25m^{10}$ .                        | 20. $25a^4 - 20a^3b^4 + 4b^8$ .       |
| 15. $16a^3 - 24a^2 + 9a^4$ .                        | 21. $a^6 - 6a^4b + 12a^2b^2 - 8b^3$ . |
| 16. $4x^4 + 20x^2 + 25$ .                           | 22. $y^3 + 6y^2z + 12yz^2 + 8z^3$ .   |
| 23. $8u^3 + 12u^2v + 6uv^2 + v^3$ .                 |                                       |
| 24. $125a^3 - 75a^2b + 15ab^2 - b^3$ .              |                                       |
| 25. $x^6 + 3x^4y^2 + 3x^2y^4 + y^6$ .               |                                       |
| 26. $729m^3 - 1215m^2n + 675m^2n^2 - 125n^3$ .      |                                       |
| 27. $b^3c^4 - 3ab^4c^4 + 3a^3b^5c^3 - a^3b^6$ .     |                                       |
| 28. $m^6n^3 + 3m^4n^2p^2q + 3m^2np^4q^2 + p^6q^3$ . |                                       |
| 29. $1000 - 300x^2 + 30x^4 - x^6$ .                 |                                       |
| 30. $216 + 540a^3 + 450a^6 + 125a^9$ .              |                                       |
| 31. $343d^6 - 294d^4 + 84d^2 - 8$ .                 |                                       |
| 32. $512z^{12} - 1728z^8 + 1944z^4 - 729$ .         |                                       |
| 33. $m^2 - n^2$ .                                   | 42. $25 - b^2x^2$ .                   |
| 34. $1 - x^2$ .                                     | 43. $49n^5 - 36m^2$ .                 |
| 35. $9 - y^2$ .                                     | 44. $144 - a^2b^2$ .                  |
| 36. $9x^2 - 4y^2$ .                                 | 45. $a^2x^3 - a^3x^2$ .               |
| 37. $9d^2y^2 - 1$ .                                 | 46. $a^2 + 2ab + b^2 - c^2$ .         |
| 38. $25x^4 - 4y^4$ .                                | 47. $a^3 - 2ab + b^2 - c^2$ .         |
| 39. $9a^2b^4 - 25a^4b^2$ .                          | 48. $x^4 + x^2y^2 + y^4$ .            |
| 40. $16x^4y^2 - 36x^2y^4$ .                         | 49. $4x^2 - 4xy + y^2 - 9z^2$ .       |
| 41. $a^4 - 9x^2$ .                                  | 50. $x^2 + 6xz + 9z^2 - 4y^2$ .       |
| 51. $a^2 + 6ac + 9c^2 - 4b^2 - 4bd - d^2$ .         |                                       |
| 52. $1 + 2x + x^2 - 4x^4 - 12x^5 - 9x^6$ .          |                                       |
| 53. $4 + 12a^3 + 9a^6 - a^4 - 2a^2d^2 - d^4$ .      |                                       |

54.  $a^4 - x^4$ .  
 55.  $m^2 + 3m + 2$ .  
 56.  $b^4 + 6b^2 + 8$ .  
 57.  $x^2 + 5x + 6$ .  
 58.  $9y^4 + 60y^2 + 96$ .  
 59.  $x^2 + 3x - 10$ .  
 60.  $9a^2 - 9a - 28$ .  
 61.  $4a^2 - 12a - 55$ .  
 62.  $16x^2 - 48x + 27$ .  
 63.  $25d^2 - 85d + 66$ .  
 64.  $m^2 + (n+2)m + 2n$ .  
 65.  $1 + 3a + b + 3ab$ .  
 66.  $16a^2 + 4a(b+1) + b$ .  
 67.  $9a^2 + 3a(4-n) - 4n$ .  
 68.  $25x^2 - 5x(a-1) - a$ .  
 69.  $9x^4 + 3x^2(p-m) - mp$ .  
 70.  $25y^2 + 5y^4(3x+2a) + 6ax$ .  
 71.  $m^6 + 2m^3(1-x) - 4x$ .  
 72.  $x^2 + y^2$ .  
 73.  $y^2 + 27x^2$ .  
 74.  $m^2 - n^2$ .  
 75.  $8 - a^2$ .  
 76.  $d^2 + 125$ .  
 77.  $a^2 - 27$ .  
 78.  $64m^6 - y^{12}$ .  
 79.  $343x^3 - 512y^2$ .  
 80.  $27m^3 - a^3$ .  
 81.  $4a^2 + 16b^2 + 16ab + 25c^2 + 4d^2 - 20cd - 20ac - 40bc + 8ad + 16bd$ .  
 82.  $9x^2 + 4y^2 - 12xy + 16z^2 + 25 - 40z + 24xz - 16yz - 30x + 20y$ .  
 83.  $4p^4 + 9x^2 + 12p^2x^2 + 4x^2y^2 + y^4 + 4xy^2 - 8p^2xy - 12x^2y - 4p^2y^2 - 6x^4y^2$ .  
 84.  $a^2 + 4b^4 + 4ab^2 + 9c^2 + 16d^2 + 24c^2d^2 + 6ac^2 + 12b^2c^2 + 8ad^2 + 16b^2d^2$ .  
 85.  $a^4 + 25b^2 - 10a^2b^2 + 4a^2 + 9b^4 - 12ab^2 + 4a^2 - 20ab^2 - 6a^2b^2 + 30b^4$ .  
 86.  $\frac{1}{4}x^4 + 16y^2 - 4x^2y + \frac{1}{2}y^4 + 36z^2 + 8y^2z^2 + \frac{3}{2}x^2y^2 - \frac{1}{2}x^2y^2 + 6x^2z^2 - 48yz^2$ .  
 87.  $\frac{1}{2}a^2 + \frac{3}{2}a^4 - \frac{3}{2}a^2 + 64b^4 + \frac{1}{4}b^2c^2 - 8b^2c - \frac{3}{2}ab^2 + 24a^2b^2 + \frac{3}{2}abc - \frac{3}{2}a^2bc$ .  
 88.  $0.36m^4 + 0.25n^2 - 0.6m^2n + 0.64p^2 + 9x^{10} - 4.8p^2x^2 + 0.96m^2p^2 - 0.8np^2 - 3.6m^2x^2 + 3nx^2$ .



122.  $a + b - c + d$ .
123.  $(x+y)^4 - (x+y)^3z + (x+y)^2z^2 - (x+y)z^3 + z^4$ .
124.  $a^6 - a^5(y+z) + a^4(y+z)^2 - a^3(y+z)^3 + a^2(y+z)^4 - a(y+z)^5 + (y+z)^6$ .
125.  $(m+n)^2 + (m+n)p + p^2$ .
126.  $(a-b)^3 - (a-b)^2x + (a-b)x^2 - x^3$ .
127.  $d^5 + d^4(e-h) + d^3(e-h)^2 + d^2(e-h)^3 + d(e-h)^4 + (e-h)^5$ .
128.  $f^3 - f^2(x-y) + f(x-y)^2 - (x-y)^3$ .
129.  $m^3n^3 + m^2n^2(x+y) + mn(x+y)^2 + (x+y)^3$ .
130.  $\frac{1}{2}a - \frac{1}{8}d$ .
131.  $\frac{1}{4}x^2 - \frac{1}{2}xy + y^2$ .
132.  $\frac{1}{81}a^4 + \frac{1}{27}a^3m + \frac{1}{9}a^2m^2 + \frac{1}{3}am^3 + m^4$ .
133.  $\frac{1}{28}d^2 - \frac{1}{20}dh + \frac{1}{16}h^2$ .
134.  $1 + \frac{2}{3}x + \frac{4}{3}x^2$ .
135.  $x^4 - x^2y + y^2$ .
136.  $a^8 + a^6m^2 + a^4m^4 + a^2m^6 + m^8$ .
137.  $y^3 - y^2z + yz^2 - z^3$ .
138.  $m^6 + m^4n^2 + m^2n^4 + n^6$ .

## Ex. 8. Page 23.

- |                                  |   |
|----------------------------------|---|
| 1. $a(a+2b)$ .                   | 4. $(x^2+8)(x^2-8)$ .   |
| 2. $(3x-1)(x+1)$ .               | 5. $\left(\frac{x}{a} - \frac{a}{x}\right)\left(\frac{x}{a} - \frac{a}{x}\right)$ . |
| 3. $(6x+7a)(6x-7a)$ .            |   |
| 6. $(x-3)(x+2)$ .                |   |
| 7. $x(x+2)(x^2+x+1)(x^2+3x+3)$ . |   |
| 8. $(x+1)(x^4-x^3+x^2-x+1)$ .    |   |
| 9. $(x+1)(x+7)$ .                |   |
| 10. $(2x+3)(2x+13)$ .            |   |



11.  $(a+b+c+d)(a+b-c-d)(a-b+c+d)(c+d-a+b)$ .
12.  $(a-d+b-c)(a-d-b+c)$ .
13.  $(x+a)(x+b)(x+c)$ .
14.  $(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$ .
15.  $(x+y)[(a^2+ab+b^2)x+(a^2-ab+b^2)]y$ .
16.  $(a+2)(a-2)(a-2b+1)$ .
17.  $[(a+b)^2+4a^2]^2-[2a(a+b)]^2$ .
18.  $(x-7)(x+4)$ .
19.  $(1-10x)(1+3x)$ .
20.  $(x-2)(x+2)(x^2+2x+4)(x^2-2x+4)$ .
21.  $3a^2(a+2b)$ .
22.  $(x-2)(x+3)$ .
23.  $(3x+8)(5x-7)$ .
24.  $(9a^2b^2+16c^2)(3ab+4c)(3ab-4c)$ .
25.  $(x+2y)(x-2y)$ .
26.  $(x^2+\frac{1}{2}x+1)(x^2-\frac{1}{2}x+1)$ .
27.  $(2x-3)(3x-2)$ .
28.  $(2x-3y)(2x+3y)(4x^2-6xy+9y^2)(4x^2+6xy+9y^2)$ .
29.  $(x+1)(x+22)$ .
30.  $(3x+5y)(3x-5y)$ .
31.  $(x-1)(x-5)$ .
32.  $(x^2+\frac{1}{2})(x^2+\frac{1}{2})$ .
33.  $(x+12)(x+12)$ .
34.  $(x-2)(x^2+2x+4)$ .
35.  $7x^4(x^2-3)$ .
36.  $x(x-12)$ .
37.  $(x^2+2)(x^2-2)$ .
38.  $(x-7)(x-9)$ .
39.  $(a+b)(c+d)$ .
40.  $(1-8x)(1+12x)$ .
41.  $(2x+y-z)(2x-y+z)$ .
42.  $(x+3)(x^2-3x+9)$ .
43.  $(ab-1)(a^2b^2+ab+1)$ .
44.  $7xy(x+y)(x^2+xy+y^2)^2$ .
45.  $-4x^2(x-16)$ .
46.  $(x^m+ay^m)(x^m+by^m)$ .

47.  $(x+ab)(x-ab)(x+2a^2)(x-2a^2)$ .
48.  $(x+3y)(x^2+3xy+9y^2)$ .
49.  $(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$ .
50.  $(x-bc-ca)(x+ca+ab)$ .
51.  $(x+y+z)(x^2+y^2+z^2-yz-zx-xy)$ .
52.  $(x-y)[(a^2+ab+b^2)x+(a^2-ab+b^2)y]$ .
53.  $(x^2+yz)(x^4-4x^2yz+7y^2z^2)$ .
54.  $(a^3-b^3-c^3)(a^3-b^3+c^3)$ .
55.  $(x^2-ab)(x-a)(x-b)$ .
56.  $(a+b+c+d)(a+b+c+d)$ .
57.  $(x^2y^2+8z)(x^2y^2+12z)$ .
58.  $(ax-by)(ac-bd)$ .
59.  $(a+b-c)(a+b-c)$ .
60.  $(x-a)(x+2a+b)$ .
61.  $(6x^2+5)(6x^2+11)$ .
62.  $3(y+z)(z+x)(x+y)$ .
63. Write the expression in the form :  

$$x^4-xy^3-xy^3+y^4=x(x^3-y^3)-y^3(x-y)$$

$$=(x-y)(x^3+x^2y+xy^2-y^3).$$
64.  $\left(x-\frac{1}{x}\right)^2-\left(\frac{1}{3}\right)^2$ .
65.  $(x^2+1)(ax+1)$ .
66.  $(x^2+4y^2)(x^{12}-4x^8y^2+16x^4y^4-64x^2y^6+256y^8)$ .
67.  $3ab(a+b)$ .
68.  $(x^2+3x-1)(x^2-3x-1)$ .
69.  $5x(x^2-2x-1)$ .
70.  $\left(x^m+\frac{1}{x^m}\right)\left(x^m+\frac{1}{x^m}\right)$ .
71.  $\left(\frac{x}{a}+\frac{a}{x}\right)\left(\frac{x}{a}+\frac{2a}{x}\right)$ .
72.  $(x^2+y^2+xy)(x^2+y^2-xy)$ .
73.  $(x-4y)(x-5y)$ .
74.  $-(x-3y)(x-3y)$ .
75.  $(a^4+9)(a^2+3)(a^2-3)$ .
76.  $a(a-4b)$ .
77.  $(x-1)(x^2+x+1)$   
 $(x^6+2x^3+3)$ .

78.  $\left(x - \frac{y}{3}\right)(3x + 10y)$ .      80.  $(x^2 + y^2)^2 - \left(\frac{3xy}{2}\right)^2$ .
79.  $(x + a)(x + b)$ .      81.  $(x + 6)(x + 20)$ .
82.  $(x^2 + x + 1)(x^2 - x + 1)$ .
83.  $(a^2b^2 + c^4)(ab^2 + c^2)(ab^2 - c^2)$ .
84.  $(x - 1)(x^2 + x + 1)(x^2 + 2)$ .
85.  $(x^2 - y^2 + 4xy)(x^2 - y^2 - 4xy)$ .
86.  $4x(x + a)$ .
87.  $(x + 11)(x + 13)$ .
88.  $-a^2(x - 3a)(x - 3a)$ .
89.  $(a - b)(a + b)(a^2 + ab + b^2)(a^2 - ab + b^2)$ .
90.  $x^4 - [(a + b)^2 + (a + c)^2]x^2 + (a + b)^2(a + c)^2$   
 $= [x^2 - (a + b)^2][x^2 - (a + c)^2]$   
 $= (x + a + b)(x - a - b)(x + a + c)(x - a - c)$ .
91.  $(x^2 + m)(ax^2 + bx - am)$ .      94.  $(2b)(2b)$ .
92.  $x^m y^{n+1}(ax^2y^2 + bxy + c)$ .      95.  $2(a - b)(1 - ab)$ .
93.  $(x - ab + bc)(x + ab + ac)$ .      96.  $(x - 3bc)(x - ca - ab)$ .
97.  $11x(x^2 + 3y)(x^2 + 16y)$ .
98.  $[(a + b)x - (a - b)y][(a - b)x + (a + b)y]$ .
99.  $(x - y)(3x^2 + 3xy + 4y^2)$ .
100.  $(1 - abc)(1 + 2abc - a^2 - b^2 - c^2)$ .
101.  $(x + y + z)(x^2 - yz)$ .
102.  $-(a + b + c)(b + c - a)(c + a - b)(a + b - c)$ .
103.  $[(a + b)x + (a - b)y][(a - b)x + (a + b)y]$ .
104.  $(ax - by)(ay - bx)$ .      107.  $2(x^5y^2 - 15)(x^5y^2 - 15)$ .
105.  $(ax + b)(bx + a)$ .      108.  $(2ax - by)(2bx - ay)$ .
106.  $(x - c)(x + a + b)$ .      109.  $(x + y)(b + c)(x + y - b - c)$ .
110.  $[x - (a - b)^2][x - (a + b)^2]$ .

111.  $(x-2y-6)(x+y+4)$ .      115.  $(a+3b)(a-b)$ .  
 112.  $a^2x(a-x)(6ax+1)$ .      116.  $(x+8)(x+9)$ .  
 113.  $2(2+5ab)(2-5ab)$ .      117.  $(x+\frac{1}{2})(x+\frac{1}{3})$ .  
 114.  $(x+6y)(x-2y)$ .      118.  $9(x^2+4)$ .  
 119.  $\left(\frac{x}{a}+\frac{a}{x}\right)^2-1$ .  
 120.  $(x^3+27)(x^3-8)$   
        $= (x+3)(x^2-3x+9)(x-2)(x^2+2x+4)$ .  
 121.  $(x-11)(x-11)$ .  
 122.  $(6x+5)(10x-9)$ .  
 123.  $(x+1)(ax+b)$ .  
 124.  $(a+b+c)(a+b-c)(c+a-b)(c-a+b)$ .  
 125.  $(4x-y)(5x-y)$ .  
 126.  $\left(x^2+\frac{10x}{13}+1\right)\left(x^2-\frac{10x}{13}+1\right)$ .  
 127.  $(x+8)(x-7)$ .  
 128.  $x(x+y)(x^2-3xy+3y^2)$ .  
 129.  $(x^3+a^3)(x^4+a^4)(x^2+a^2)(x+a)(x-a)$ .  
 130.  $(7x+9y^2)(7x-9y^2)$ .      139.  $(a+b)(a^2+b^2)$ .  
 131.  $(x^2-5)(x^2-8)$ .      140.  $(x^3+125)(x^3-125)$ .  
 132.  $(x+\frac{1}{2})(x+\frac{1}{3})$ .      141.  $(x+a)(x-b)(x+c)$ .  
 133.  $(x-11)(x+12)$ .      142.  $-2a(x+y)$ .  
 134.  $x(x-10)$ .      143.  $(a+b+c)(a+b+c)$ .  
 135.  $4x^2y^2(y+2x)(y-2x)$ .      144.  $(4xy+3)^2(4xy-3)^2$ .  
 136.  $3(1+2x)(1-2x)$ .      145.  $(3x^2+3xy+2y^2)$   
        $(3x^2-3xy+2y^2)$ .  
 137.  $x^2(x-a^3)(x-a^2)$ .  
 138.  $(2x-3)(2x+1)$ .      146.  $(x+b)(x+a+b)$ .

147.  $(x+1)(ax^2+bx+a)$ .  
148.  $(a+b)(x-a)(x^2+ax+a^2)$ .  
149.  $-(a+b+c)(b+c-a)(c+a-b)(a+b-c)$ .  
150.  $(x+y)[(a+b)x-3by]$ .  
151.  $4(a-2)(a+2)(b+4)(b-4)$ .  
152.  $(x-y-z)(x^2+y^2+z^2-yz+zx+xy)$ .  
153.  $(a-x)(a+x)(b+x)$ .  
154.  $(bcx-a^2)(bcx-1)$ .  
155.  $(2x+3y)(2x-3y)^2$ .  
156.  $(x+2a)(x-a-b)$ .  
157.  $(ax+by)(by+c)$ .  
158.  $5x^2(x+3y)(x+14y)$ .  
159.  $(x-a)(x-b-c)$ .  
160.  $(5x^{11}+4y^2)(16x^{11}-15y^2)$ .  
161.  $\left(1+\frac{x}{2}+a+\frac{b}{2}\right)\left(1+\frac{x}{2}-a-\frac{b}{2}\right)$ .  
162.  $(a-bx)(a+bx+cx^2)$ .  
163.  $(x-a+3)(x-a-2)$ .  
164.  $x(x+y)$ .  
165.  $(x^2+yz)(x^4-4x^2yz+7y^2z^2)$ .  
166.  $3(x^2+5y)(x^2+5y)$ .  
167.  $(x-3)(x+1)$ .  
168.  $(x^2-3x+1)(x^2+3x+1)$ .  
169.  $(4x^2+9y^2)(2x+3y)(2x-3y)$ .  
170.  $x^4(x^2+4a^2)(x+2a)(x-2a)$ .  
171.  $(x^2+4xy+8y^2)(x^2-4xy+8y^2)$ .  
172.  $(x^2-11)(x^2+10)$ .  
173.  $2(4-3x)(2+7x)$ .  
174.  $(2x-1)(x+1)$ .  
175.  $x^2y(a+3y)(a^2-3ay+9y^2)$ .

176.  $x(x-1)(7x+103)$ .      178.  $(1+x)(1+14x)$ .  
 177.  $(x-2)(x-4)$ .      179.  $(x+6)(x+9)$ .  
 180.  $(x^2-4x+1)(x^2+4x+1)$ .  
 181.  $\left(x^2+\frac{5x}{19}+1\right)\left(x^2-\frac{5x}{19}+1\right)$ .  
 182.  $(x-a-2)(x+a+1)$ .      189.  $4ab$ .  
 183.  $21(x-1)(3x+2)$ .      190.  $(x+5y)(x+6y)$ .  
 184.  $(2x-1)(x^2-2)$ .      191.  $(3x-5y)(3x+14y)$ .  
 185.  $(x+y)(x^2+y^2)$ .      192.  $(x-a^2+b^2)(x+a^2-b^2)$ .  
 186.  $3(x-7)(x+2)$ .      193.  $(a+b+c)(a+b+c)$ .  
 187.  $5x^2(3-2x)(5+x)$ .      194.  $(ax+by)(bx-ay)$ .  
 188.  $(x-10)(x-20)$ .      195.  $(x+1)(2x^4+3x^2+4)$ .  
 196.  $(x+a)(x+2b)$ .  
 197.  $(a+b)(x+a)(x^2-ax+a^2)$ .  
 198.  $(x^2+4ax-4a^2)(x-2a)^2$ .  
 199.  $2(x+2a)(x+3b)$ .  
 200.  $(x^2-c^2-2b^2)(x^2+c^2-2a^2)$ .  
 201.  $3\left(1-\frac{a}{3}+x\right)\left(1-\frac{a}{3}-x\right)$ .  
 202.  $(ax+b)(bx-a)$ .  
 203.  $2b(x+y)$ .  
 204.  $(ax+by)(bx+ay)$ .  
 205.  $[a^2-(b-c)^2][a^2-(b+c)^2]$ .  
 206.  $(x-a)^2(x+b)$ .  
 207.  $(x-2y+3z)(x-2y-3z)$ .  
 208.  $(4x+5b)(2x+3a)$ .  
 209.  $3(3a-3b-4x)(9a-9b+5x)$ .  
 210.  $(x-1)(x+1)(x^2+2x+2)$ .  
 211.  $2(2x^2-5y^2)(x^2+10y^2)$ .

212.  $(x + 2a)(x + 3b)$ .  
 213.  $(a + 2b + c)(2b + c - a)(c + a - 2b)(a + 2b - c)$ .  
 214.  $3x^2y^2z^2(x^2y - 2y^2z + 4xz^2)$ . 227.  $(x + a + b)(x + a - b)$ .  
 215.  $(x - 2a)(x - b)$ . 228.  $5(x - 1)(x + 1)$ .  
 216.  $7(2x - 1)(2x - 5)$ . 229.  $(x - y)(x + 10y)$ .  
 217.  $(x^2 - 3xy + y^2)(x^2 + 3xy + y^2)$ .  
 218.  $(x^2 - 13x - 1)(x^2 + 13x - 1)$ .  
 219.  $(a + b)^2(a - b)^2$ . 230.  $(7xy - 1)(4xy + 1)$ .  
 220.  $3ab(b - a)$ . 231.  $(x + 2)(x^2 + 4)$ .  
 221.  $(x - 12)(x + 25)$ . 232.  $4abxy$ .  
 222.  $5(1 - 7x^2y^2)(1 - 7x^2y^2)$ . 233.  $(7x + y)(7x + 6y)$ .  
 223.  $\frac{5}{9}(3x - 10y)(3x + y)$ . 234.  $(1 + ab)(1 + 18ab)$ .  
 224.  $(x^2 + 13x + 1)(x^2 - 13x + 1)$ .  
 225.  $(x+1)(x-1)(x+3)(x-3)$ . 235.  $(x + 1)(ax - b)$ .  
 226.  $3y^2(x - y)(3x + 2y)$ . 236.  $(x - 2y)(x - 6y)$ .  
 237.  $(x^2 + y^2)(x + y)(x - y)(x^2 + 9y^2)(x + 3y)(x - 3y)$ .  
 238.  $(x^2 - 20)(x^2 + 31)$ . 242.  $(3x + 5y)(7x - 3y)$ .  
 239.  $(a^2 + b^2)(a - 2b)$ . 243.  $(x - ca - ab)(x + bc + ca)$ .  
 240.  $14(2x - 3)(x + 2)$ . 244.  $(ab - c^2)(ac - b^2)$ .  
 241.  $7(x^5 - 1)(x^5 - 2)$ . 245.  $(x^2 - 6y^2)(13x^2 - 40y^2)$ .  
 246.  $(ax + y)(x + ay)$ .  
 247.  $(x^2 + mxy + y^2)(x^2 - mxy + y^2)$ .  
 248.  $x(x - a)(x - b)$ .  
 249.  $(x + 7)(x - 2)(x + 2)(x + 3)$ .  
 250.  $[x^2 + (1 + m)y^2][x^2 + (1 - m)y^2]$ .  
 251.  $(9x^2 - 11y)(11x^2 + 12y)$ .  
 252.  $(x^2y - z^2)(x^2y - 16z^2)$ .  
 253.  $(x - ab - 4c)(x + ab - 3c)$ .  
 254.  $(x^2 + mxy - y^2)(x^2 - mxy - y^2)$ .  
 255.  $(ax - by)(bx - ay)$ .

256.  $a(a-2b+3x)(a-2b-3x)$ .  
 257.  $(ax^2+bx+c)(ax^2-bx+c)$ .  
 258.  $(x-a)(x+a)(x-b)(x+b)$ .  
 259.  $(x+yz)(y+zx)$ .  
 260.  $(x+y-10z)(x+y+3z)$ .  
 261.  $(ax-c)(bx+c)$ .  
 262.  $(x+1)(x+2)(x+4)(x+5)$ .  
 263.  $3x(3x+5)$ .  
 264.  $(x^2-a^2-ab-b^2)(x^2+a^2+b^2)$ .  
 265.  $(5x^2-3y^2)(9x^2+25y^2)$ . 273.  $(x^2+y^2)(x+8y)(x-8y)$ .  
 266.  $(2a-b)(a^2+2bx)$ . 274.  $(x-3y^2)(11x-21y^2)$ .  
 267.  $(x-a+b)(x+a+b)$ . 275.  $(x^2+4y)(x^2+9y)$ .  
 268.  $(x-b+a)(x-b-a)$ . 276.  $3(x+2y)(x+15y)$ .  
 269.  $11(3x-4)(3x-5)$ . 277.  $(a-b)(a^2-2b^2)$ .  
 270.  $ab(a^2-2ab-2b^2)$ . 278.  $\frac{4}{15}(4a-b)(4b-a)$ .  
 271.  $(x^4y^2+3)(x^4y^2+12)$ . 279.  $x^2(6a+x)(4a-3x)$ .  
 272.  $12(4x^2-11)(4x^2+1)$ . 280.  $(3x+2)(x^2-7)$ .  
 281.  $a^3(a^2+ab-b^2)(a^2-ab+b^2)$ .  
 282.  $(ax+y)(bx-y)$ .  
 283.  $(x-b)(x+b)(x-a^2)$ .  
 284.  $(a^4-b^4)(a^2+b^2)^2(a+b)^2(a-b)^2$ .  
 285.  $(x-a)(x+a)(x+b)$ .  
 286.  $(x+y)(x-y)^3$ .  
 287.  $(2+ax)(a-2x)$ .  
 288.  $5(x^4-25)+4x(x^2-5)$   
 $= (x^2-5)(5x^2+4x+25)$ .  
 289.  $(a+bx)(a-bx+cx^2)$ .  
 290.  $x^2(6x-7)(6x+5)$ .  
 291.  $2a(a^2+3b^2)$ .



## Ex. 9. Page 31.

- |                                  |                      |                                  |
|----------------------------------|----------------------|----------------------------------|
| 1. $2(x+1)$ .                    | 25. $3x-2$ .         | 47. $a^2+b^2$ .                  |
| 2. $9(a+b)$ .                    | 26. $3x-2a$ .        | 48. $a(2a^2+2ab-b^2)$ .          |
| 3. $3(x+1)$ .                    | 27. $2x+3$ .         | 49. $7x^2+8x+1$ .                |
| 4. $4(x^2+y^2)$ .                | 28. $x-1$ .          | 50. $x^4-2x^2+3x^2$<br>$-2x+1$ . |
| 5. $a-x$ .                       | 29. $x(x+3)$ .       | 51. $x^2-3x+1$ .                 |
| 6. $ax(a-x)^2$ .                 | 30. $2(7-3x)$ .      | 52. $x+1$ .                      |
| 7. $a^2-b^2$ .                   | 31. $x+1$ .          | 53. $a(2a-3x)$ .                 |
| 8. $x-y$ .                       | 32. $x^2+7$ .        | 54. $(x+1)^3$ .                  |
| 9. $3(a^2c+2b^2)$ .              | 33. $x-y$ .          | 55. $2x^3-4x^2+x-1$ .            |
| 10. $(2a+3b)$ .                  | 34. $a+5$ .          | 56. $x-2a$ .                     |
| 11. $b-a$ .                      | 35. $x-y$ .          | 57. $x+a$ .                      |
| 12. $2a+5$ .                     | 36. $a-b$ .          | 58. $x^2-(a+b)x+ab$ .            |
| 13. $4a+3b$ .                    | 37. $x-a$ .          | 59. $2x-9$ .                     |
| 14. $5x-1$ .                     | 38. $(x-y)^2$ .      | 60. $x^2-1$ .                    |
| 15. $x^5(3a^3-4ax^2y^2-6x^2y)$ . | 39. $5x^2-1$ .       | 61. $x^2-(a-b)x+b^2$ .           |
| 16. $x+y$ .                      | 40. $x^2-4x+4$ .     | 62. $x^3+x^2+x+1$ .              |
| 17. $(a+b)^2$ .                  | 41. $x^2-4y^2$ .     | 63. $x+1$ .                      |
| 18. $2x-3y$ .                    | 42. $a(a^2-2a+1)$ .  | 64. $x+a$ .                      |
| 19. $a+b$ .                      | 43. $9x^2(x-1)$ .    | 65. $a-b$ .                      |
| 20. $x+5$ .                      | 44. $x^2-y^2$ .      | 66. $x-2a$ .                     |
| 21. $x-7$ .                      | 45. $a^3+a^2-5a+3$ . | 67. $a+b$ .                      |
| 22. $x-10$ .                     | 46. $x+3$ .          | 68. $3x-y$ .                     |
| 23. $x-12$ .                     |                      |                                  |
| 24. $x+1$ .                      |                      |                                  |

## Ex. 10. Page 34.

- |                     |                       |                          |
|---------------------|-----------------------|--------------------------|
| 1. $100a^5b^4c^6$ . | 4. $90b(a^2-b^2)$ .   | 7. $12(a^2-x^2)$ .       |
| 2. $ab(a+b)$ .      | 5. $(a+b)(c^2-d^2)$ . | 8. $ab(a^2x^2-b^2y^2)$ . |
| 3. $12ab(b-1)$ .    | 6. $(a+b)(a-b)^2$ .   | 9. $b^2(b+ax)$ .         |

10.  $(x+1)(x+3)(x+4)$ . 12.  $12x^2 - 8x^2 - 27x + 18$ .  
 11.  $x^3 + 3x^2 - x - 3$ . 13.  $x(2x+1)(3x-1)(4x+3)$ .  
 14.  $(x+2)(x+4)(x^2 + 3x + 1)$ .  
 15.  $36x^3 - 3ax^2 - 50a^2x + 24a^3$ , or  $(4x-3a)(3x-2a)(3x+4a)$ .  
 16.  $(a^3 - 9a^2 + 23a - 15)(a-7)$ .  
 17.  $(a^2 - b^2)(a^2 - 4b^2)$ .  
 18.  $60x^6y^3 + 190x^5y^3 + 116x^4y^4 + 64x^3y^5 + 48x^2y^6 - 30xy^7$ .  
 19.  $a^6 - b^6$ . 28.  $x^3 - 1$ .  
 20.  $mnpq$ . 29.  $(x+1)(x+2)(x+3)$ .  
 21.  $36x^3y^3x^2$ . 30.  $(x^3-19x-30)(x^2+5x+10)$ .  
 22.  $x^3 - y^3$ . 31.  $(x+1)(x+2)(x+3)(x-1)$ .  
 23.  $(x+y)(x-y)^2$ . 32.  $(4x^3-9)(9x^2-4)$ .  
 24.  $6a(a^2+3a+2)$ . 33.  $(a-1)(a-2)(a-3)(a-4)$ .  
 25.  $a(a^2-4b^2)$ . 34.  $x^8 - 1$ .  
 26.  $a(4a^2-1)$ . 35.  $x^{12} - 1$ .  
 27.  $2a(a^2-b^2)$ . 36.  $(x-a)(x-b)(x-c)$ .  
 37.  $(x+c)(2x-3b)(x^2+ax-b^2)$ .

## Ex. 11. Page 35.

1.  $\frac{3n^3}{2q}$ . 6.  $\frac{4ab}{9x}$ . 11.  $\frac{b^2-3ac}{c^2-4ab}$ .  
 2.  $\frac{2a}{3b}$ . 7.  $\frac{7b^2-4ac}{6c^2-7ab}$ . 12.  $\frac{2x^2-y}{3x^2-y^2}$ .  
 3.  $\frac{3b^3}{5ac}$ . 8.  $-\frac{8a^2b-x^3}{b^2x-9a^3}$ . 13.  $\frac{2z^2-3y^2}{3z^2-2y^2}$ .  
 4.  $\frac{6a^2}{5x^2}$ . 9.  $-\frac{2x^4-3a^2b^2}{4a^4-3bx^3}$ . 14.  $\frac{3c^2-a^2}{2a^2+c^2}$ .  
 5.  $\frac{4a}{7b^2c^2}$ . 10.  $\frac{a+b}{a(1-3c^2)}$ . 15.  $\frac{3a^2-7c}{4a^2+9b^2}$ .

- |                                      |                                     |                                |
|--------------------------------------|-------------------------------------|--------------------------------|
| 16. $\frac{5a^2 - 3b}{2(2b^2 - 3a)}$ | 31. $\frac{3a}{5a - 9b}$            | 47. $\frac{x+4}{x+6}$          |
| 17. $\frac{1}{2x+1}$                 | 32. $\frac{(a-b)(a^2+b^2)}{a+b}$    | 48. $\frac{a-7b}{a+b}$         |
| 18. $\frac{1}{3x-1}$                 | 33. $\frac{x+z}{(1-y)^2}$           | 49. $\frac{a+b}{a-b}$          |
| 19. $\frac{1}{8x^2-1}$               | 34. $\frac{ax}{(a+b)^2}$            | 50. $\frac{x+3}{x-3}$          |
| 20. $\frac{x^2}{x-3}$                | 35. $\frac{4a^3x^2}{3b(5a^2+4b)}$   | 51. $\frac{2x-3}{7x-3}$        |
| 21. $\frac{xy}{x+1}$                 | 36. $\frac{x^2-ax+b^2}{x^2+ax-b^2}$ | 52. $\frac{4a-3b}{5a+2b}$      |
| 22. $\frac{x^2-y}{x}$                | 37. $\frac{x+c}{a+b-x}$             | 53. $\frac{2x-1}{x-1}$         |
| 23. $\frac{x(3x^2+2)}{2y^2}$         | 38. $\frac{x-a}{x-b+c}$             | 54. $\frac{3x-4y}{5x-3y}$      |
| 24. $\frac{x^2y^2}{x^2x^2+y^4}$      | 39. $\frac{x-b}{bc-a(c+x)}$         | 55. $\frac{4y+3}{3(y+2)}$      |
| 25. $\frac{3x}{4(a-3x)}$             | 40. $\frac{bc}{(a+b+c)(b+c-a)}$     |                                |
| 26. $\frac{x^2}{x+y}$                | 41. $\frac{x+y+z}{x-y-z}$           | 56. $\frac{(2a+1)x}{a+1}$      |
| 27. $\frac{x^2}{x+2}$                | 42. $\frac{x+2}{x+5}$               | 57. $\frac{(x-1)^2}{x^2-3x+1}$ |
| 28. $\frac{4x}{3x+2y}$               | 43. $\frac{x+7}{x-5}$               | 58. $\frac{a^2+b^2}{a}$        |
| 29. $\frac{3b^2}{a^2-2c}$            | 44. $\frac{x-2}{x-3}$               | 59. $\frac{1}{b-2x}$           |
| 30. $\frac{1}{5x(7x^2+3)}$           | 45. $\frac{x-2}{x+1}$               | 60. $\frac{7(x^2+xy+y^2)}{5}$  |
|                                      | 46. $\frac{x-4}{x-5}$               |                                |

## Ex. 12. Page 38.

1.  $\frac{a+2m^2}{m}$
2.  $\frac{3x-a}{x-a}$
3.  $\frac{4b}{3a+2b}$
4.  $\frac{3a^2-2b^2}{6ab}$
5.  $\frac{5ay-2bx}{x^2y}$
6.  $\frac{x^2-2y^2}{10xy}$
7.  $\frac{5n+ax}{5a^2}$
8. 2.
9.  $\frac{b^2-a^2}{ab}$
10.  $\frac{3a+b}{2b}$
11.  $\frac{4a+3b}{12a}$
12.  $\frac{21a-16b}{6}$
13.  $\frac{133a}{36}$
14.  $\frac{2a+3b}{b}$
15.  $\frac{x+y}{3}$
16.  $\frac{a-b}{2}$
17.  $-\frac{x+5y}{15}$
18.  $\frac{a}{12}$
19. 0.
20.  $\frac{4a}{45}$
21.  $\frac{3b+5c-5a}{36}$
22.  $\frac{11a-8b+3c}{84}$
23. 0.
24.  $a+b$ .
25.  $\frac{5a+286b}{30b}$
26.  $\frac{ab+ac+bc}{abc}$
27.  $\frac{a(np+mp+mn)}{mnp}$
28.  $\frac{az-by-cx}{xyz}$
29.  $\frac{2ab+3bx-4ax}{abx^2}$
30.  $\frac{7}{60b}$
31.  $\frac{a}{6x}$
32.  $\frac{a(3df-5cf+4cd)}{bcd f}$
33.  $\frac{2(9adg+60ad-5cg)}{45bdg}$
34.  $\frac{a}{18x}$
35.  $\frac{ab+ac+bc}{abc}$
36.  $\frac{a^2+b^2+c^2}{abc}$
37.  $\frac{a^2-x^2}{2a^2x^2}$
38.  $\frac{2a}{a^2-b^2}$
39.  $\frac{a^2+2ab-b^2}{a^2-b^2}$

40.  $\frac{2bc}{a^3-b^3}$       47.  $\frac{3x+2}{x-1}$       54.  $\frac{2a+b}{2a-b}$   
 41.  $\frac{1}{x-y}$       48.  $\frac{6ax}{a-x}$       55.  $\frac{2}{x(1-4x^2)}$   
 42.  $\frac{12a}{1-9a^2}$       49.  $\frac{2xa^3}{a^3-1}$       56.  $\frac{16}{(x-2)(x+2)^2}$   
 43.  $\frac{a+x}{ax}$       50.  $\frac{4a}{a+x}$       57.  $\frac{a^4+6a^2x^2+x^4}{a^4-x^4}$   
 44.  $\frac{a+b}{2(a-b)}$       51.  $\frac{b(a+b)}{x^2-b^2}$       58.  $\frac{2}{(x+1)(x+2)(x+3)}$   
 45.  $\frac{6x}{2a-3x}$       52.  $\frac{b}{a-b}$       59.  $\frac{x(5x-7)}{(x^2-1)(x-2)}$   
 46.  $\frac{3a^2-2ab+3b^2}{2(a^2-b^2)}$       53.  $\frac{2x-3}{x(4x^2-1)}$       60.  $\frac{2x-3}{(x^2-1)(2x+3)}$   
 61.  $\frac{8x^4-x^3-24x^2+5x-4}{(x^2-1)(x+3)(x-2)}$   
 62.  $\frac{3a^4+4a^3x+6a^2x^2-4ax^3+3x^4}{a^4-x^4}$   
 63.  $\frac{2x^3+21x+13}{(1-x^2)(1-4x^2)}$   
 64.  $\frac{2}{x^4-y^4}$       72.  $-1$       79.  $\frac{x(x-2)}{x^3+1}$   
 65.  $\frac{18(2a+15)}{16a^4-81}$       73.  $\frac{1}{abc}$       80.  $\frac{2x^2}{x^3+y^3}$   
 66.  $\frac{x+c}{(x-a)(b-x)}$       74.  $0$       81.  $\frac{2y^2}{x^3-y^3}$   
 67.  $0$       75.  $\frac{2x^2-9x+44}{x^3+64}$       82.  $\frac{4-3x+2x^2-3x^4}{2(1+x^2)(1+x^3)}$   
 68.  $0$       76.  $\frac{2a}{x^2-a^2}$       83.  $\frac{2(x^3+1)}{x^4+x^3+1}$   
 69.  $\frac{x^3-4ax-a^3}{(x^2-a^2)^2}$       77.  $\frac{x^3+1}{x^4-3x^3+3x^2-x}$       84.  $\frac{3x^3+9}{x^3-27}$   
 70.  $0$       78.  $1$   
 71.  $0$

85.  $\frac{4a^3}{a^3 + a^2 + 1}$ . 86. 0. 91.  $\frac{4}{x+2}$ .
87.  $\frac{2(4-x-3x^3)}{(x-1)^3}$ . 92.  $\frac{5a}{x-5a}$ .
88.  $-\frac{2x^4+10x^3-9x^2-16x+4}{(4-x^2)^3}$ . 93.  $\frac{a(x-9a)}{(x-6a)(x-4a)}$ .
89.  $\frac{9x^3+1}{(2x-1)^3}$ . 94.  $\frac{12+25x-x^3}{(3+x^2)(3+x)^2}$ .
90.  $\frac{7a^3-4ax-2x^3}{2(a-2x)^3}$ . 95.  $\frac{84-186x+93x^2-6x^3}{4(3-2x)^2(3+2x)}$ .
96.  $\frac{(z^4-2z^2-3)(4z^2-3)-(z^2-4z+1)(5z^4+5z^2-9)}{(4z^2-3)(3z^2+2)(5z^4+5z^2-9)}$ .
97.  $\frac{4x^3+40x^2+70x-22}{x^4+5x^3-13x^2-77x-60}$ .
98.  $\frac{11-x^2}{(x-2)(x-5)(x-7)}$ . 99.  $\frac{3a^3-41ax+74x^3}{3(2a-5x)^3}$ .
100. 0.

## Ex. 13. Page 43.

1.  $\frac{18p^3q^3r^3}{35m^2x^{11}y^{13}}$ . 7.  $\frac{3a^9}{5bc}$ . 13.  $\frac{8a^6b^9x^{13}}{27y^{15}}$ .
2.  $\frac{8x^3}{3yz^3}$ . 8.  $\frac{2m^{12}n}{a^3b^{25}}$ . 14.  $\frac{a^0y^{5p}x^{5m-15}}{32b^{5q-5}z^{20}}$ .
3.  $\frac{4d}{9acq}$ . 9.  $\frac{a^2}{b^2}$ . 15.  $\frac{1}{9a^5x^2}$ .
4. 1. 10.  $\frac{8a^3}{125b^3}$ . 16.  $\frac{9a^{10}c^6x}{2b^{11}}$ .
5.  $\frac{ab^3}{y^2}$ . 11.  $\frac{625x^4y^4}{2401a^4b^4}$ . 17.  $\frac{16b^3}{27a^3}$ .
6.  $\frac{3c^2}{8}$ . 12.  $\frac{32a^5b^5}{243c^5x^5}$ . 18.  $\frac{2(a-b)}{3b}$ .

19.  $\frac{4x(x-y)}{x^2+y}$       24.  $\frac{a^2}{d^2}$       28.  $\frac{9}{2x}$   
 20.  $\frac{25b^8c^{10}}{1024a^{20}}$       25.  $\frac{20}{21}$       29.  $\frac{y(x-y)}{x(x+y)}$   
 21.  $\frac{2}{5}$       22.  $\frac{4y}{x-1}$       26.  $\frac{12ab}{5f(a-b)}$       30.  $\frac{x^2+y^2}{x}$   
 23.  $-\frac{3x^2(x-y)}{4(x^2+y^2)}$       27.  $\frac{(3x-1)}{x-2}$       31.  $\frac{ax}{a^2-x^2}$   
 32.  $\frac{(a^3+b^3)(a+b)}{(a^2+b^2)(a^2-ab+b^2)}$       34.  $\frac{a^2-ab+b^2}{a^2+ab+b^2}$   
 33.  $\frac{x+p}{x+n}$       35.  $\frac{a+b}{a-b}$   
 36.  $\frac{a(2a+5x)(2a^2+19ax+42x^2)}{x(a-x)(a^2+2ax+2x^2)}$   
 37.  $\frac{(a+b)^2}{ab}$ , or  $2+\frac{b}{a}+\frac{a}{b}$       46.  $\frac{x^2}{a^2}+\frac{a^2}{x^2}-\frac{y^2}{b^2}-\frac{b^2}{y^2}$   
 38.  $2b+\frac{3b^2}{a}-\frac{2a^2}{b}-3a$       47.  $\frac{4ab}{c^2x^3y^5}$   
 39.  $\frac{1}{a^2}-\frac{1}{b^2}$       48.  $\frac{15a^m x^n}{28b^m y^n}$   
 40.  $\frac{ad}{bc}-\frac{145}{72}+\frac{bc}{ad}$       49.  $\frac{7b^2x^{n-3}}{3a^m c^2}$   
 41.  $\frac{x}{x-y}$       50.  $\frac{a^3 z^7}{b^5 y^4}$   
 42.  $\frac{4ab}{a^2-b^2}$       51.  $\frac{xy^2 z^3}{a^p b^m c^n}$   
 43.  $\frac{a^2(a-b)}{x}$       52.  $\frac{21pqxy}{5mnzu}$   
 44.  $\frac{(a-c)^2-b^2}{abc}$       53.  $\frac{77abxy}{15mnpq}$   
 45.  $\frac{x^8-ax^5+a^5x-a^8}{a^3x^3}$       54.  $\frac{6a^8b^8c^3}{65d^9f^9g^{14}}$

- |                                       |                                  |                              |
|---------------------------------------|----------------------------------|------------------------------|
| 55. $\frac{55q^4y^2}{42p^3r^2x^3z^4}$ | 62. $\frac{2}{a-b}$              | 68. $\frac{x+y}{a-b}$        |
| 56. $-1$                              | 63. $\frac{x+y}{a+b}$            | 69. $-\frac{y^2}{m^2}$       |
| 57. $-\frac{2}{3}$                    | 64. $\frac{y-b}{y-a}$            | 70. $\frac{a-b}{b}$          |
| 58. $\frac{1}{3(x-y)}$                | 65. $\frac{x+y-z}{x-y+z}$        | 71. $\frac{b(a-x)}{a(a-b)}$  |
| 59. $\frac{3(a-b)^2}{b}$              | 66. $\frac{1}{a^2-b^2}$          | 72. $-\frac{a(x+2)}{c(x+1)}$ |
| 60. $\frac{20b(y+z)}{3(x-y)}$         | 67. $\frac{(a-2)^2}{(a-3)(a-1)}$ | 74. $\frac{p^2+3}{p-3}$      |
| 61. $\frac{x(a+2x)}{a^2}$             |                                  |                              |
73.  $\frac{8m^2(4m-3n)(2m-9n)}{27n^2(2m+9n)(4m+3n)}$

## Ex. 14. Page 47.

- |                         |                           |                                   |
|-------------------------|---------------------------|-----------------------------------|
| 1. $\frac{x+y}{z}$      | 8. $\frac{a+1}{a-1}$      | 16. 1.                            |
| 2. $\frac{m-c}{a+d}$    | 9. $\frac{x+y}{x-y}$      | 17. $\frac{a-4}{a-5}$             |
| 3. $\frac{3mn-mp}{5np}$ | 10. $\frac{m+n}{m-n}$     | 18. $\frac{1}{x+1}$               |
| 4. $-\frac{5n}{12}$     | 11. $\frac{2xy}{x^2+y^2}$ | 19. $\frac{y^2-a^2}{y(a+b+c)-bc}$ |
| 5. $\frac{3x-2y}{3z-u}$ | 12. $\frac{mq+np}{oq-nt}$ | 20. $\frac{1}{a+1}$               |
| 6. $\frac{x}{z}$        | 13. $\frac{y}{x}$         | 21. $\frac{1+a}{1+a^2}$           |
| 7. $-\frac{d}{c}$       | 14. $y$                   | 22. $a+1$                         |
|                         | 15. 1.                    | 23. $\frac{1+a^2}{1+a}$           |



24.  $\frac{4}{3(a+1)}$

25.  $\frac{adf+ae}{bdf+be+cf}$

26. 1.

27.  $\frac{bc+ac+ab}{bc+ac-ab}$

28.  $-\frac{m^4+m^2n^2+n^4}{mn(m-n)^2}$

29.  $-\frac{xy(y-x)^2}{x^4+y^4+x^2y^2}$

30.  $\frac{ab}{a^2+b^2}$

31.  $\frac{(m^2+n^2)^2}{2m^2n^2}$

32.  $a$ .

33.  $\frac{4xy^3}{x^4-y^4}$

34.  $\frac{(a+b+c)^2}{2bc}$

35.  $\frac{12m}{5n}$

36. 3.

37. 1.

38.  $\frac{a^2-b^2}{16a^2b^2}$

39.  $\frac{x-y}{xy}$

40.  $\frac{4}{3}$

41.  $\frac{2(1+x^4)}{x}$

42.  $\frac{3+x^2}{x(1-x^2)}$

43. 1.

## Ex. 15. Page 51.

1.  $\frac{0}{a}=0$ .

2.  $\frac{1}{\infty}=0$ .

3.  $\frac{n}{0}=\infty$ .

4.  $\frac{0}{0}$

5.  $\frac{0}{0}$

6.  $\frac{3}{7}$

7.  $\frac{1}{2}$

8.  $\frac{3}{4}$

9. -1.

10. 32.

11.  $\frac{2}{3}$

12. 0.

13.  $\infty$ .

14.  $\frac{3}{5}$

15. 0.

16.  $\frac{3a}{2}$

17.  $\frac{1}{4}$  and  $\frac{2}{5}$

18. 4.

19. 4.

20.  $\frac{4}{5}$

21.  $\frac{9}{8}$

22.  $\frac{3}{2}$

23.  $\frac{7}{12}$

24.  $\frac{7}{5}$

25.  $\frac{1}{8}$

26.  $\frac{11}{17}$

27.  $\frac{1}{2}$

28.  $-\frac{1}{2}$

29. 5.

30.  $-\frac{a^2-24}{3a^2+16}$

31.  $\frac{a}{b}$

32.  $\frac{3}{2}$

## Ex. 16. Page 53.

- |                        |                         |   |                           |
|------------------------|-------------------------|---|---------------------------|
| 1. $\frac{m^3}{n^3}$ . | 4. 0.                   | 8. $b^3$ .                                    | 11. $\frac{1-x^3}{1+x^3}$ |
| 2. $\frac{n}{m}$ .     | 5. $a$ .                | 9. 2.   | 12. 0.                    |
| 3. 0.                  | 6. 0.                   | 10. $\frac{a}{m-\frac{1}{b}} = \frac{a}{m}$ . | 13. 1.                    |
|                        | 7. $\frac{2x}{1+x^2}$ . |   |                           |

$$19. \frac{(a+b-c)(a-b+c)}{2bc}.$$

$$20. x^{n-1}; \text{ remaining fraction, } \frac{x^n}{1-x}.$$

$$21. \frac{1}{0.97} = \frac{1}{1-0.03} = 1 + 0.03 + 0.0009 + 0.000027 \\ + 0.00000081 = 1.0309278.$$

$$22. \frac{3}{107} = \frac{3}{100+7} = 0.03 - 0.0021 + 0.000147 \\ - 0.00001029 + 0.00000072 = 0.0280374.$$

$$23. \frac{3}{104} = \frac{3}{100+4} = 0.03 - 0.0012 + 0.000048 \\ - 0.00000192 = 0.0288461.$$

$$24. \frac{5}{0.98} = 5 \left( \frac{1}{1-0.02} \right) = 5(1 + 0.02 + 0.0004 + 0.000008 \\ + 0.00000016) = 5.10204080.$$

$$25. \frac{7}{0.995} = 7 \left( \frac{1}{1-0.005} \right) = 7(1 + 0.005 + 0.000025 + \dots) \\ = 7.0351758793968.$$

$$26. \frac{3}{1001} = 0.00299700299700.$$

$$27. 0.8787\dots$$

## Ex. 17. Page 56.

1. -12.	29. 70.	57. 11.	85. 13.
2. $-\frac{2}{11}$ .	30. 12.	58. 5.	86. 12.
3. $-52\frac{3}{40}$ .	31. 28.	59. 28.	87. 3.
4. 0.	32. 36.	60. 7.	88. $\frac{45}{228}$ .
5. 1.	33. 20.	61. $\frac{4}{18}$ .	89. $2\frac{1}{2}$ .
6. $\frac{1}{4}$ .	34. 4.	62. 4.	90. $2\frac{1}{2}$ .
7. 4.	35. 6.	63. 17.	91. $\frac{35}{7}$ .
8. 5.	36. 11.	64. $84\frac{1111}{1111}$ .	92. $\frac{6}{18}$ .
9. 3.	37. 1.	65. $-\frac{427}{1789}$ .	93. 2.
10. 2.	38. -5.	66. 1.	94. -7.
11. 7.	39. $7\frac{117}{117}$ .	67. $-22\frac{1}{4}$ .	95. 6.
12. 3.	40. 8.	68. $10\frac{125}{444}$ .	96. 7.
13. 1.	41. 2.	69. -6.	97. 17.
14. 5.	42. 3.	70. $\frac{101}{101}$ .	98. 6.
15. 7.	43. 12.	71. $\frac{5}{18}$ .	99. 2.
16. $2\frac{2}{3}$ .	44. $\frac{1}{7}$ .	72. 8.	100. 0.
17. 4.	45. 10.	73. 7.	101. 2.
18. 0.	46. 5.	74. 36.	102. 4.
19. 8.	47. 6.	75. 6.	103. $\frac{1}{2}$ .
20. $-1\frac{76}{116}$ .	48. 1.	76. $-\frac{2111}{1111}$ .	104. 8.
21. 1.	49. 19.	77. $1\frac{117}{117}$ .	105. 2.
22. 2.	50. $1\frac{57}{885}$ .	78. 2.43466.....	106. 1.
23. 1.	51. $3\frac{5}{21}$ .	79. 6.	107. $\frac{33}{34}$ .
24. 10.	52. 10.	80. 36.	108. 7.
25. $-1\frac{11}{11}$ .	53. $-10\frac{17}{17}$ .	81. 3.	109. 7.
26. 90.	54. 13.	82. 24.	110. 3.
27. 28.	55. $23\frac{5}{8}$ .	83. 4.	
28. 12.	56. 3.	84. 17.	

## Ex. 18. Page 62.

- |   |   |                                     |
|---|---|-------------------------------------|
| 1. $\frac{k-m}{p(n-q)}$                         | 7. $\frac{ab}{c}$                         | 12. $\frac{4m-3n}{5}$               |
| 2. $\frac{db-ag}{bg(c-3)}$                      | 8. $\frac{bm-cn-dn}{b}$                   | 13. 0.                              |
| 3. $\frac{2d}{a+b}$                             | 9. $\frac{1-m}{1+m}$                      | 14. 0.                              |
| 4. $\frac{q(m+n)-p(c-d)}{pq}$                   |   | 15. $\frac{2m}{n}$                  |
| 5. $\frac{m(m-n+1)}{m+n+1}$                     | 10. $\frac{bp-m}{n-bq}$                   | 16. $\frac{np}{m}$                  |
| 6. $\frac{p}{m}$                                | 11. $\frac{n(a+b)}{m(a-b)}$               | 17. 1.                              |
|   |   | 18. 1.                              |
|   |   | 19. 0.                              |
| 20. $\frac{a^2-b^2}{b-4a}$                      | 33. $\frac{0.28bc^2d^2}{d^2-0.2bc}$       |                                     |
| 21. $\frac{abcd}{ab+ac+bc}$                     | 34. $\frac{2b^3+12ab^2-9a^3}{6a^2+10b^2}$ |                                     |
| 22. $\frac{bdgh}{adg-bcg+ddf}$                  | 35. $\frac{5(a^2+b^2)-15a^3}{2(10a-3b)}$  |                                     |
| 23. $\frac{a^2bc^2+ab+c^2}{a^2-bc}$             | 36. $\frac{n}{m}$                         | 37. $\frac{mq}{anp}$                |
| 24. $\frac{c(-a^3b+a^2+ab^2+ab-b^2)}{ab+ac-bc}$ | 38. $\frac{cfn+bdp}{adp-cfm}$             |                                     |
| 25. $\frac{b^2+c^2-ac+b^2c}{b+c}$               | 39. $\frac{b}{b-a-1}$                     |                                     |
| 26. $\frac{15a-4b+3abc}{a-5b+abd}$              | 40. $\frac{mq(m+n)}{p}$                   |                                     |
| 27. 0.  | 30. 1.                                    | 41. $\frac{n(n^2+m^2)}{m(n^2-m^2)}$ |
| 28. 0.  | 31. $\frac{a^2b}{a-c}$                    | 42. $\frac{d}{c}$                   |
| 29. n.  | 32. a-b.                                  |                                     |

$$43. \frac{a(b-c) + 3(b+c)}{b+c}.$$

$$44. mn + mp + np.$$

$$45. \frac{mnp}{m+n+p}.$$

$$46. \frac{3a(a-b)}{8b-3a}.$$

$$47. \frac{2(15a^2 + 17ab + 4b^2)}{25a}.$$

$$48. \frac{3(8a^2 + 6ab - b^2)}{17a - 6b}.$$

$$49. 5a.$$

$$50. 4a.$$

$$51. \frac{3d^2 - 2c^2}{12cd}.$$

$$52. \frac{n}{n-1}.$$

$$53. \frac{b(a-b+c)}{a}.$$

$$54. \frac{a^2(b-a)}{b(b+a)}.$$

$$55. \frac{c^2 - ab}{a+b-2c}.$$

$$56. \frac{mn}{m+n-p}.$$

$$57. \frac{mn(m+n-2p)}{(m+n)p - m^2 - n^2}.$$

$$58. \frac{2mn}{m+n}.$$

$$59. \frac{a+b+c+d}{m+n}.$$

$$60. \frac{c^2}{d-c}.$$

$$61. \frac{ab}{a+b}.$$

$$62. 5a.$$

$$63. \frac{5a(a+b)}{2(a+4b)}.$$

$$64. \frac{81m(n-m)}{56n}.$$

$$65. \frac{a-3b}{3}.$$

$$66. \frac{a}{6}.$$

$$67. m+n.$$

$$68. \frac{1}{p} + \frac{1}{q}.$$

$$69. p.$$

$$70. q.$$

$$71. \frac{p(s-m)}{pn+pr-qs}.$$

$$72. \frac{de}{d+e}.$$

$$73. 4k.$$

$$74. \frac{f(4f^2-9g^2)}{8f^3+27g^3}.$$

$$75. \frac{4a^2(ab-b^2+a^2)}{3a^3-6a^2b+ab^3+6b^3}.$$

## Ex. 19. Page 66.

1. 160,000 inhabitants.
2. Together,  $27\frac{2}{3}$  dys.; A alone,  $62\frac{1}{3}$  dys.; B alone,  $110\frac{1}{3}$  dys.; C alone,  $38\frac{1}{3}$  dys.
3.  $2.58\frac{1}{2}$  lbs.
4. 4.55 ft.
5. 32.110 yds.
6. 2 hrs. 24 min.
7.  $\frac{bm}{a-b}$  hrs.
8. 1188 mi.
9. 30 mi. per hour.
10. 4 mi.
11. The next day at 1.30 o'clock in the morning, and at 188 mi. from A.
12. At the end of 6 hrs., and at  $31\frac{1}{2}$  mi. from A.
13.  $32\frac{8}{11}$  min. past 12.
14. 10 times;  $5\frac{5}{11}$  min. past 1;  $10\frac{1}{11}$  min. past 2;  $16\frac{4}{11}$  min. past 3; and so on.
15. 300 leaps.
16. 6780 steps.
17. Length of the route 15 mi.; speed, 2 mi. per hour.
18.  $4\frac{4}{5}$  mi. per hour.
19.  $\frac{n(t'-t)}{t'(m+1)}$ .
20. Together,  $\frac{2mnp}{mn+mp+np}$  dys.; A alone,  $\frac{2mnp}{mp+np-mn}$  dys.; B alone,  $\frac{2mnp}{mn+np-mp}$  dys.; C alone,  $\frac{2mnp}{mn+mp-np}$  dys.
21.  $\frac{abn}{b-a}$  ft.
22.  $\frac{pqr}{pq+q-p}$
23.  $\frac{ka}{n(n-d-a)}$  miles;  $16\frac{2}{3}$  mi.
24. A's part =  $\frac{ads}{ad+bd+bh}$ ; B's part =  $\frac{bds}{ad+bd+bh}$ ; C's part =  $\frac{bhs}{ad+bd+bh}$ .
25.  $\frac{d(b-d)}{d+1}$ .

26. Of the first  $\frac{l(m-b)}{a-b}$  gals.; of the second,  $\frac{l(a-m)}{a-b}$  gals.

27.  $\frac{4a}{\pi}$

28.  $\frac{100^{\circ}A}{(100+t)^{\circ}}$

29. A's part =  $\frac{amg}{am+bn+cp}$ ; B's part =  $\frac{bng}{am+bn+cp}$ ;  
C's part =  $\frac{cpg}{am+bn+cp}$ .

30.  $\frac{an}{(n+1)^2} - n$ ;  $\frac{an}{(n+1)^2} + n$ ;  $\frac{a}{(n+1)^2}$ ;  $\frac{an^2}{(n+1)^2}$ ;  
18, 22, 10, 40.

31.  $\frac{2mn(2m+n)}{4m^2+4mn-n^2}$  dys. 32.  $\frac{abmp}{mp-(p-n)a}$  33.  $\frac{6b}{b-ac}$

Ex. 20. Page 71.

- |                         |   |                                   |  |
|-------------------------|---|-----------------------------------|--|
| 1. $x=4$ ,<br>$y=3$ .   | 11. $x=-2$ ,<br>$y=19$ .                    | 21. $x=1$ ,<br>$y=-1$ .           | 31. $x=6$ ,<br>$y=12$ .                            |
| 2. $x=12$ ,<br>$y=4$ .  | 12. $x=-5$ ,<br>$y=14$ .                    | 22. $x=34$ ,<br>$y=46$ .          | 32. $x=1$ ,<br>$y=2$ .                             |
| 3. $x=8$ ,<br>$y=4$ .   | 13. $x=-3$ ,<br>$y=-2$ .                    | 23. $x=5$ ,<br>$y=4\frac{1}{2}$ . | 33. $x=28$ ,<br>$y=49$ .                           |
| 4. $x=13$ ,<br>$y=3$ .  | 14. $x=7$ ,<br>$y=-5$ .                     | 24. $x=1$ ,<br>$y=\frac{1}{2}$ .  | 34. $x=6$ ,<br>$y=12$ .                            |
| 5. $x=7$ ,<br>$y=2$ .   | 15. $x=\frac{1}{2}$ ,<br>$y=-\frac{1}{8}$ . | 25. $x=-2$ ,<br>$y=-3$ .          | 35. $x=10$ ,<br>$y=5$ .                            |
| 6. $x=5$ ,<br>$y=1$ .   | 16. $x=-2$ ,<br>$y=1$ .                     | 26. $x=13$ ,<br>$y=5$ .           | 36. $x=20$ ,<br>$y=20$ .                           |
| 7. $x=7$ ,<br>$y=17$ .  | 17. $x=3$ ,<br>$y=4$ .                      | 27. $x=10$ ,<br>$y=4$ .           | 37. $x=9$ ,<br>$y=7$ .                             |
| 8. $x=12$ ,<br>$y=9$ .  | 18. $x=2$ ,<br>$y=1$ .                      | 28. $x=12$ ,<br>$y=5$ .           | 38. $x=12$ ,<br>$y=6$ .                            |
| 9. $x=2$ ,<br>$y=3$ .   | 19. $x=2$ ,<br>$y=3$ .                      | 29. $x=9$ ,<br>$y=8$ .            | 39. $x=\frac{1}{4}$ ,<br>$y=\frac{1}{5}$ .         |
| 10. $x=3$ ,<br>$y=20$ . | 20. $x=8$ ,<br>$y=2$ .                      | 30. $x=24$ ,<br>$y=45$ .          | 40. $x=\frac{1067}{286}$ ,<br>$y=\frac{278}{59}$ . |

41.  $x = 6$ ,  $y = 5$ .  
 42.  $x = 19\frac{1}{2}$ ,  $y = -17$ .  
 43.  $x = 3$ ,  $y = 2$ .  
 44.  $x = 11$ ,  $y = 13$ .  
 45.  $x = 3$ ,  $y = 4$ .  
 46.  $x = 5$ ,  $y = 6$ .  
 47.  $x = \frac{1}{2}$ ,  $y = \frac{1}{4}$ .  
 48.  $x = 3$ ,  $y = 4$ .  
 49.  $x = \frac{1}{2}$ ,  $y = \frac{1}{4}$ .  
 50.  $x = \frac{9}{2}$ ,  $y = \frac{61}{108}$ .  
 51.  $x = \frac{1}{8}$ ,  $y = \frac{1}{8}$ .  
 52.  $x = \frac{1}{8}$ ,  $y = \frac{1}{4}$ .  
 53.  $x = \frac{1}{2}$ ,  $y = \frac{1}{8}$ .  
 54.  $x = 0.7$ ,  $y = 0.3$ .  
 55.  $x = 2$ ,  $y = 3$ .  
 56.  $x = 21$ ,  $y = 20$ .  
 57.  $x = 7\frac{8}{9}$ ,  $y = 12\frac{8}{9}$ .  
 58.  $x = 9$ ,  $y = 7$ .  
 59.  $x = 8\frac{8}{9}$ ,  $y = 8\frac{1}{4}$ .  
 60.  $x = 7$ ,  $y = 5$ .  
 61.  $x = \frac{8}{9}$ ,  $y = -\frac{3}{2}$ .  
 62.  $x = 4$ ,  $y = 7$ .  
 63.  $x = -\frac{3}{8}$ ,  $y = \frac{1}{8}$ .  
 64.  $x = 3$ ,  $y = 2$ .  
 65.  $x = 2$ ,  $y = 3$ .  
 66.  $x = 3$ ,  $y = 2$ .  
 67.  $x = 2$ ,  $y = 2$ .

68. We first obtain  $\frac{xy-5}{xy-4} = \frac{xy-7}{xy-8}$ , from which we find

$xy = 6$ . The second equation gives  $xy = x - 1$ .  
 Introducing the value of  $xy = 6$ , we find  $x = 7$ , and  
 substituting,  $y = \frac{6}{7}$ .

### Ex. 21. Page 76.

1.  $x = \frac{a+b}{2}$ ,  $y = \frac{a-b}{2}$ .  
 2.  $x = \frac{5(a-b)}{2}$ ,  $y = \frac{a+b}{2}$ .  
 3.  $x = a+b$ ,  $y = a-b$ .  
 4.  $x = a^2+ab+b^2$ ,  $y = a^2-ab+b^2$ .  
 5.  $x = \frac{cq-bx}{cq-bp}$ ,  $y = \frac{cp-ar}{bp-aq}$ .  
 6.  $x = \frac{em+bn}{ae+bc}$ ,  $y = \frac{an-cm}{ae+bc}$ .  
 7.  $x = \frac{n'r+nr'}{mn'+m'n}$ ,  $y = \frac{mr'-m'r}{mn'+m'n}$ .  
 8.  $x = \frac{1}{ab}$ ,  $y = \frac{1}{cd}$ .  
 9.  $x = \frac{2b^2-6a^2+d}{3a}$ ,  $y = \frac{3a^2-b^2+d}{3b}$ .  
 10.  $x = \frac{a(bf+g)}{c+f}$ ,  $y = \frac{g-bc}{c+f}$ .



$$11. x = \frac{abcd}{a + bd},$$

$$y = \frac{abc}{a + bd}.$$

$$12. x = \frac{abce}{bg - af},$$

$$y = \frac{abcf}{bg - af}.$$

$$13. x = \frac{mr(np - qs)}{nr - ms},$$

$$y = \frac{ns(rq - mp)}{nr - ms}.$$

$$14. x = \frac{n(qtr + prs)}{ps + mqr},$$

$$y = \frac{qs(t - mv)}{ps + mqr}.$$

$$15. x = \frac{bp(dfnt - cgps)}{gt(adnp - bcmq)},$$

$$y = \frac{dg(agps - bfmt)}{gt(adnp - bcmq)}.$$

$$16. x = \frac{(ab + ac - bc)abc}{a^2b^2 + a^2c^2 - b^2c^2},$$

$$y = \frac{(ac - ab - bc)abc}{a^2b^2 + a^2c^2 - b^2c^2}.$$

$$17. x = \frac{m}{m - n},$$

$$y = \frac{n}{m + n}.$$

$$18. x = -\frac{(a - m)(a - n)}{a - b},$$

$$y = \frac{(b - m)(b - n)}{a - b}.$$

$$19. x = \frac{a^2 - b^2}{a},$$

$$y = \frac{a^2 - b^2}{b}.$$

$$20. x = m + n,$$

$$y = m - n.$$

$$21. x = (p^2 + 1)(q^2 - 1),$$

$$y = (p^2 - 1)(q^2 + 1).$$

$$22. x = \frac{1 + m}{1 + mn},$$

$$y = \frac{m(1 + n)}{1 + mn}.$$

$$23. x = \frac{m^2q}{mq - np},$$

$$y = \frac{mnq}{mq - np}.$$

$$24. x = \frac{s - mq - np}{2q},$$

$$y = \frac{s + mq + np}{2p}.$$

$$25. x = \frac{(d^2 - c^2)b - (a^2 - b^2)c}{bd - ac},$$

$$y = \frac{(a^2 - b^2)d - (d^2 - c^2)a}{bd - ac}.$$

$$26. x = \frac{2}{m + n},$$

$$y = \frac{2}{m - n}.$$

$$27. x = \frac{m - n}{p - q},$$

$$y = \frac{m - n}{mq - np}.$$

$$28. x = \frac{ms - nr}{ps - nt},$$

$$y = \frac{ms - nr}{mt - pr}.$$

$$29. x = \frac{mnpr}{mq + nr},$$

$$y = \frac{mnpq}{mq + nr}.$$

$$30. x = \frac{a}{b},$$

$$y = \frac{c}{d}.$$

$$31. x = a + b - c,$$

$$y = a - b + c.$$

$$32. x = \frac{m^2 + mn + n^2}{m + n},$$

$$y = \frac{m^2 - mn + n^2}{m - n}.$$

$$33. x = \frac{m + n}{p},$$

$$y = \frac{m - n}{p}.$$

$$34. x = \frac{p + 1}{mp - 1},$$

$$y = \frac{m + 1}{mp - 1}.$$

$$35. x = a + b - c,$$

$$y = a - b + c.$$

$$36. x = \frac{a + 1}{b},$$

$$y = \frac{b + 1}{a}.$$

$$37. x = \frac{a - b}{a + b},$$

$$y = \frac{a + b}{a - b}.$$

$$38. x = \frac{bm}{b - m},$$

$$y = \frac{bm}{b + m}.$$

$$39. x = \frac{p + q}{p - q},$$

$$y = \frac{p - q}{p + q}.$$

$$40. x = \frac{m}{m + n},$$

$$y = \frac{n}{m - n}.$$

$$41. x = \frac{a + b}{a},$$

$$y = \frac{a - b}{b}.$$

$$42. x = \frac{m + n}{3n},$$

$$y = \frac{n - m}{2m}.$$

$$43. x = \frac{pr + q}{r},$$

$$y = \frac{pq - r}{q}.$$

$$44. x = \frac{pq + m}{q},$$

$$y = \frac{pq - m}{p}.$$

$$45. \begin{aligned} x &= 5(c+d), \\ y &= 3(c-d). \end{aligned}$$

$$50. \begin{aligned} x &= \frac{c-d}{c+d}, \\ y &= \frac{c+d}{c-d}. \end{aligned}$$

$$46. \begin{aligned} x &= a(a+1) + b(b-1), \\ y &= a(a-1) - b(b-1). \end{aligned}$$

$$51. \begin{aligned} x &= \frac{m+n^2}{2n}, \\ y &= \frac{m-n^2}{2n}. \end{aligned}$$

$$47. \begin{aligned} x &= a - 2b + 3c, \\ y &= 3a - 2b + c. \end{aligned}$$

$$52. \begin{aligned} x &= \frac{m^2+n^2}{2n}, \\ y &= \frac{m^2-n^2}{2n}. \end{aligned}$$

$$48. \begin{aligned} x &= m + 3n + 2p, \\ y &= 3m - n + 2p. \end{aligned}$$

$$49. \begin{aligned} x &= \frac{m^2-n^2}{m}, \\ y &= \frac{m^2-n^2}{n}. \end{aligned}$$

$$53. \begin{aligned} x &= c^2 - d^2, \\ y &= c^2 + d^2. \end{aligned}$$

$$54. \begin{aligned} x &= a^5 + b^5, \\ y &= a^5 - b^5. \end{aligned}$$

## Ex. 22. Page 80.

$$1. \begin{aligned} x &= 2, \\ y &= 3, \\ z &= 4. \end{aligned}$$

$$6. \begin{aligned} x &= 10, \\ y &= 20, \\ z &= 30. \end{aligned}$$

$$11. \begin{aligned} x &= 3, \\ y &= 2, \\ z &= 1. \end{aligned}$$

$$16. \begin{aligned} x &= 4, \\ y &= 6, \\ z &= 8. \end{aligned}$$

$$2. \begin{aligned} x &= 1, \\ y &= 3, \\ z &= 5. \end{aligned}$$

$$7. \begin{aligned} x &= 1.7, \\ y &= 1.5, \\ z &= 1.3. \end{aligned}$$

$$12. \begin{aligned} x &= 6, \\ y &= 8, \\ z &= 10. \end{aligned}$$

$$17. \begin{aligned} x &= 8, \\ y &= 6, \\ z &= 2. \end{aligned}$$

$$3. \begin{aligned} x &= 3, \\ y &= 5, \\ z &= 7. \end{aligned}$$

$$8. \begin{aligned} x &= 11, \\ y &= 7, \\ z &= 9. \end{aligned}$$

$$13. \begin{aligned} x &= 11, \\ y &= 12, \\ z &= 13. \end{aligned}$$

$$18. \begin{aligned} x &= 1\frac{3}{8}, \\ y &= 4\frac{5}{8}, \\ z &= 7\frac{3}{8}. \end{aligned}$$

$$4. \begin{aligned} x &= 1, \\ y &= 2, \\ z &= 3. \end{aligned}$$

$$9. \begin{aligned} x &= 12, \\ y &= 18, \\ z &= 35. \end{aligned}$$

$$14. \begin{aligned} x &= 12, \\ y &= 34, \\ z &= 56. \end{aligned}$$

$$19. \begin{aligned} x &= -1.2, \\ y &= 3.4, \\ z &= -5.6. \end{aligned}$$

$$5. \begin{aligned} x &= 2, \\ y &= 4, \\ z &= 8. \end{aligned}$$

$$10. \begin{aligned} x &= 15, \\ y &= 6, \\ z &= 8. \end{aligned}$$

$$15. \begin{aligned} x &= 50, \\ y &= 31, \\ z &= 19. \end{aligned}$$

$$20. \begin{aligned} x &= 20, \\ y &= 10, \\ z &= 5. \end{aligned}$$

- |   |  |  |  |
|---|--|--|--|
| 21. $x = 15,$<br>$y = 12,$<br>$z = 10.$                     | 32. $x = 10,$<br>$y = 2,$<br>$z = 3.$      | 43. $x = 3,$<br>$y = 4,$<br>$z = 5.$       | 54. $x = 3,$<br>$y = 2,$<br>$z = 4.$   |
| 22. $x = 1,$<br>$y = 2,$<br>$z = 3.$                        | 33. $x = 4,$<br>$y = 3,$<br>$z = 5.$       | 44. $x = 10,$<br>$y = 7,$<br>$z = 3.$      | 55. $x = \frac{4}{3},$<br>$y = 4,$<br>$z = \frac{4}{3}.$                       |
| 23. $x = \frac{3}{2},$<br>$y = -7,$<br>$z = 36\frac{1}{2}.$ | 34. $x = 3,$<br>$y = 7,$<br>$z = 4.$       | 45. $x = 51,$<br>$y = 76,$<br>$z = 1.$     | 56. $x = 1\frac{1}{2},$<br>$y = -3\frac{1}{2},$<br>$z = 2\frac{1}{10}.$        |
| 24. $x = 7,$<br>$y = 8,$<br>$z = 9.$                        | 35. $x = 13,$<br>$y = 24,$<br>$z = 62.$    | 46. $x = 9,$<br>$y = 10,$<br>$z = 11, .$   | 57. $x = \frac{1}{2},$<br>$y = \frac{3}{4},$<br>$z = \frac{3}{4}.$             |
| 25. $x = 13,$<br>$y = 65,$<br>$z = 91.$                     | 36. $x = 1,$<br>$y = 2,$<br>$z = 3.$       | 47. $x = 2,$<br>$y = 3,$<br>$z = 4.$       | 58. $x = \frac{1}{2},$<br>$y = \frac{1}{3},$<br>$z = \frac{1}{4}.$             |
| 26. $x = 12,$<br>$y = 18,$<br>$z = 24.$                     | 37. $x = 10,$<br>$y = 20,$<br>$z = 30.$    | 48. $x = 0,$<br>$y = 1,$<br>$z = 3.$       | 59. $x = \frac{1}{2},$<br>$y = \frac{1}{3},$<br>$z = \frac{1}{4}.$             |
| 27. $x = 3,$<br>$y = 4,$<br>$z = 5.$                        | 38. $x = 1,$<br>$y = 2,$<br>$z = 3.$       | 49. $x = 1,$<br>$y = 2,$<br>$z = 3.$       | 60. $x = 5,$<br>$y = 4,$<br>$z = 3.$   |
| 28. $x = 1,$<br>$y = 2,$<br>$z = 3.$                        | 39. $x = 24,$<br>$y = 60,$<br>$z = 120.$   | 50. $x = 111,$<br>$y = 222,$<br>$z = 333.$ | 61. $x = 1,$<br>$y = 4,$<br>$z = \frac{1}{2},$<br>$u = 1\frac{1}{2}.$          |
| 29. $x = 5,$<br>$y = 6,$<br>$z = 8.$                        | 40. $x = 315,$<br>$y = 630,$<br>$z = 945.$ | 51. $x = 5,$<br>$y = 7,$<br>$z = 3.$       | 62. $x = 100,$<br>$y = 60,$<br>$z = -13,$<br>$u = -50.$                        |
| 30. $x = 1,$<br>$y = 4,$<br>$z = 6.$                        | 41. $x = 2,$<br>$y = 2,$<br>$z = 2.$       | 52. $x = 11,$<br>$y = 3,$<br>$z = 7.$      | 63. $x = \frac{1}{2},$<br>$y = \frac{1}{2},$<br>$z = \frac{1}{2},$<br>$u = 0.$ |
| 31. $x = 2,$<br>$y = 9,$<br>$z = 10.$                       | 42. $x = 0,$<br>$y = 1,$<br>$z = 2.$       | 53. $x = 3,$<br>$y = 7,$<br>$z = 11.$      | 64. $x = 12,$<br>$y = 5,$<br>$z = 7,$<br>$u = 4.$                              |

$$65. x = 3, y = 1, z = 5, u = 9.$$

$$66. x = 12, y = 30, z = 168, u = 50.$$

## Ex. 23. Page 86.

$$1. \begin{aligned} x &= \frac{1}{2}(a + b - c), \\ y &= \frac{1}{2}(a + c - b), \\ z &= \frac{1}{2}(b + c - a). \end{aligned}$$

$$2. \begin{aligned} x &= \frac{m + p - t}{m - q}, \\ y &= \frac{mt - mp - nq}{m - q}, \\ z &= \frac{mt - nq - pq}{m - q}. \end{aligned}$$

$$3. \begin{aligned} x &= \frac{n + p}{2}, \\ y &= \frac{m - n}{2}, \\ z &= \frac{m - p}{2}. \end{aligned}$$

$$4. \begin{aligned} x &= -abc, \\ y &= ac + ab + bc, \\ z &= -(a + b + c). \end{aligned}$$

$$5. \begin{aligned} x &= b + c, \\ y &= a + c, \\ z &= a + b. \end{aligned}$$

$$6. \begin{aligned} x &= \frac{p}{2 - m - n + mn}, \\ y &= \frac{p(m - 1)}{2 - m - n + mn}, \\ z &= \frac{p(1 - m - n + mn)}{2 - m - n + mn}. \end{aligned}$$

$$7. \begin{aligned} x &= \frac{amp}{mp + np + nq}, \\ y &= \frac{anp}{mp + np + nq}, \\ z &= \frac{anq}{mp + np + nq}. \end{aligned}$$

$$8. \begin{aligned} x &= \frac{2}{m + n - p}, \\ y &= \frac{2}{m - n + p}, \\ z &= \frac{2}{n + p - m}. \end{aligned}$$

$$9. \begin{aligned} x &= \frac{2a}{m + n}, \\ y &= \frac{2b}{m + p}, \\ z &= \frac{2c}{n + p}. \end{aligned}$$

$$10. \begin{aligned} x &= a, \\ y &= b, \\ z &= c. \end{aligned}$$

$$11. \begin{aligned} x &= \frac{mp - mn + np}{2mnp}, \\ y &= \frac{mn + np - mp}{2mnp}, \\ z &= \frac{mn + mp - np}{2mnp}. \end{aligned}$$

$$12. \quad x = \frac{2abc}{ac + bc - ab},$$

$$y = \frac{2abc}{ab + bc - ac},$$

$$z = \frac{2abc}{ab + ac - bc}.$$

$$13. \quad x = \frac{abc}{ab + bc + ac},$$

$$y = \frac{abc}{ab + bc + ac},$$

$$z = \frac{abc}{ab + bc + ac}.$$

$$14. \quad x = m^2 - n^2,$$

$$y = n^2 - p^2,$$

$$z = p^2 - m^2.$$

$$15. \quad x = (m + n)(n - p),$$

$$y = (n + p)(p - m),$$

$$z = (p + m)(m - n).$$

$$16. \quad x = n^2 - p^2,$$

$$y = p^2 - m^2,$$

$$z = m^2 - n^2.$$

$$17. \quad x = \frac{1}{n - p},$$

$$y = \frac{1}{p - m},$$

$$z = \frac{1}{m - n}.$$

$$18. \quad x = n + p - m,$$

$$y = m + p - n,$$

$$z = m + n - p.$$

$$19. \quad x = \frac{1}{2}(n + p),$$

$$y = \frac{1}{2}(p + m),$$

$$z = \frac{1}{2}(m + n).$$

20. Placing

$$s = \frac{1}{2}(m + n + p + q)$$

we have

$$x = s - m,$$

$$y = s - n,$$

$$z = s - p,$$

$$u = s - q.$$

Putting

$$s = \frac{1}{2}(l + m + n + p + q)$$

we have

$$x = s - q,$$

$$y = s - p,$$

$$z = s,$$

$$u = s - m,$$

$$v = s - n,$$

$$w = s - l.$$

$$21. \quad x = p + n,$$

$$y = q - p,$$

$$z = m + p - q,$$

$$u = q + n - m,$$

$$v = m - n.$$

$$22. \quad x = m + n + p,$$

$$y = n - m - p,$$

$$z = 3m - n + p,$$

$$u = m + n + p.$$

**Ex. 24. Page 38.**

- |  |  |
|--|--|
| 1. The first, \$3.60;<br>the second, \$3.                  | 23. Length, 15 <sup>m</sup> ; width, 10 <sup>m</sup> .                 |
| 2. 8 and 4.  | 24. Standard of the first,<br>0.800; standard of the<br>second, 0.900. |
| 3. A stitched copy, \$3;<br>a bound copy, \$3.80.          | 25. 10 gals. of the first;<br>4 gals. of the second.                   |
| 4. \$1050, the first;<br>\$600, the second.                | 26. The first, \$900;<br>the second, \$400.                            |
| 5. $\frac{2}{3}$ .                      6. $\frac{5}{7}$ . | 27. 12 men; 12 women.  |
| 7. 120 and 90.   | 28. A's library, 2400 vols.;<br>B's library, 1800 vols.                |
| 8. 42 and 12.  | 29. Velvet, \$12.40;<br>silk, \$1.60.                                  |
| 9. 63.                      12. 36.                        | 30. M, \$22,000; N, \$16,000;<br>P, \$18,000.                          |
| 10. 26.                    13. 75.                         | 31. First, 140 eggs;<br>second, 90 eggs;<br>third, 130 eggs.           |
| 11. 54.                    14. 18 and 8.                   | 32. Wheat, \$2.80; rye, \$1.80;<br>oats, \$1.                          |
| 15. 857 and 142.   | 33. 854.   |
| 16. 5 and 6.   | 34. 50, 65, and 75.  |
| 17. 25 and 55.   | 35. 640, 720, and 840.   |
| 18. 1.234 and 5.678.                                       |  |
| 19. 4 and 2 $\frac{1}{2}$ .                                |  |
| 20. Butter, 90 cts. per kilo;<br>soap, 75 cts. per kilo.   |  |
| 21. An artificer, \$4;<br>a laborer, \$2.                  |  |
| 22. 30 mi.   |  |

**Ex. 25. Page 93.**

- |               |                |               |
|---------------|----------------|---------------|
| 1. 18.055472. | 7. 30.577769.  | 13. 1.914854. |
| 2. 5.385164.  | 8. 29.017236.  | 14. 1.77240.  |
| 3. 22.15851.  | 9. 26.81417.   | 15. 1.14687.  |
| 4. 26.944387. | 10. 12.04159.  | 16. 10.955.   |
| 5. 19.773720. | 11. 3.05670.   | 17. 11.959.   |
| 6. 31.937438. | 12. 5.2145871. | 18. 60.0167.  |

19. 20.025.  
 20. 10.05.  
 21. 100.02.  
 22. 99.985.  
 23. 300.0066.  
 24. 1200.0037.  
 25. 899.9972.  
 26. 79.975.  
 27. 200.0075.  
 28.  $5x - 2 + 3y$ .  
 29.  $2x^3 - 3y^3 + 4z^5$ .  
 30.  $2 - a + 3a^2$ .  
 31.  $ab + a - b$ .  
 32.  $x^2 - 4x + 1$ .  
 33.  $3a^2 - 5b + 8m - z$ .  
 34.  $y^5 - 2u^3 + 3v^4 - 4q^5$ .  
 35.  $a^2 - 3c^5 - 4d^{22} + 3e^2$ .  
 36.  $3a^3 - 2a^2b - 7ab^2 + 4b^3$ .  
 37.  $\frac{m^2}{2} - \frac{n^3}{3} + \frac{p^4}{4}$ .  
 38.  $\frac{x}{y} - \frac{2z}{u} + \frac{3q}{v}$ .  
 39.  $\frac{3}{5} - \frac{a^4}{5} - \frac{m^3}{6} + \frac{2n^4}{7}$ .  
 40.  $\frac{3m^3n^2}{5p^3q^4} - \frac{2m^2n^3}{7p^4q^5} - \frac{4mn^4}{9p^5q^6}$ .  
 41.  $\frac{3y}{4x} - 5 + \frac{2x}{7y}$ .  
 42.  $\frac{x^7}{a^3} - x^4 - a^4$ .  
 43.  $x^2y^3 - \frac{y^4}{3a^3} + \frac{2y^{15}}{a^9}$ .  
 44. 9.535.  
 45. 10.7462.  
 46. 9.920.  
 47. 8.0259.  
 48. 11.78554.  
 49. 17.002.  
 50. 20.7792.  
 51. 24.368.  
 52. 38.3565.  
 53. 27.550.  
 54. 1.4816.  
 55. 6.8243.  
 56. 2.4887.  
 57. 8.8365.  
 58. 21.5515.  
 59.  $a - 3y + 2b$ .  
 60.  $7x - 3u + v$ .  
 61.  $5a^3 - 2b^2 + c^4$ .  
 62.  $2x^5 + 3y^2 - 2z^3$ .  
 63.  $3a^4 - b^5 + 6d^7$ .  
 64.  $10m^2 - 3n - 2p$ .  
 65.  $2a^2b + 3ab^2 - 2b^3$ .  
 66.  $5x^2y - 2x^2y^3 + 3xy$ .  
 67.  $6m^2n^3 - 4m^3n^4 + 5mn^2$ .  
 68.  $3ax^2 + 2a^2x^3 - 5a^3x^4$ .  
 69.  $\frac{2a^2}{x} - \frac{3a^3}{x^2} + \frac{a^4}{x^3}$ .  
 70.  $\frac{m^3}{y^5} + \frac{m^2}{y^4} - \frac{m}{y^3}$ .  
 71.  $\frac{5x}{y^2} - \frac{x^2}{2y^3} + \frac{4x^3}{y^4}$ .  
 72.  $\frac{2}{a^2} + \frac{1}{a^3} - \frac{5}{a^4}$ .  
 73.  $\frac{3a}{4b} - \frac{2b}{3a^2} + \frac{5a^3}{2b^2}$ .  
 74.  $\frac{2a}{x^2} + \frac{3a^2}{x^3} + \frac{4a^3}{x^4}$ .  
 75.  $3b^2 - 4b^3 - 5b^4$ .



## Ex. 26. Page 97.

- |   |                                   |  |               |
|---|-----------------------------------|--|---------------|
| 1. $\pm 2$ .                            | 5. $\pm \sqrt{\frac{5}{2}}$ .     | 11. $\pm 9$ .                                  | 12. $\pm 3$ . |
| 2. $\pm \frac{\sqrt{85}}{2} = 4.6097$ . | 6. $\pm 9$ .                      | 13. $\pm \sqrt{-4}$ .                          |               |
| 3. $\pm 5$ .                            | 7. $\pm 4$ .                      | 14. $\pm \sqrt{-(m+n)^2}$ .                    |               |
| 4. $\pm \frac{7}{\sqrt{2}}$ .           | 8. $\pm \frac{1}{2}\sqrt{-144}$ . | 15. $\pm \sqrt{a^2b^2 - 2b^2}$ .               |               |
| 17. $\pm \sqrt{a^2 + b^2 - c^2}$ .      | 9. $\pm 6$ .                      | 16. $\pm \sqrt{a^2b^2 - b^4}$ .                |               |
| 18. $\pm \sqrt{a^2 + 6ab + b^2}$ .      | 10. $\pm \frac{2}{3}$ .           | 22. $\pm \sqrt{\frac{11a^2}{2} - 3ab + b^2}$ . |               |
| 19. $\pm 2m$ .                          |                                   | 23. $\pm \sqrt{(a+b)^2 + (3a)^2}$ ; or,        |               |
| 20. $\pm \sqrt{-pn}$ .                  |                                   | $\pm \sqrt{10a^2 + 2ab + b^2}$ .               |               |
| 21. $\pm b$ .                           |                                   | 24. $\pm \sqrt{a^2 + 34ab + b^2}$ .            |               |
|   |                                   | 25. $\pm (a+b)$ .                              |               |

## Ex. 27. Page 98.

- |                                       |                                     |                                       |
|---------------------------------------|-------------------------------------|---------------------------------------|
| 1. 8; -6.                             | 15. 4; 24.                          | 29. 1; $\frac{2}{3}$ .                |
| 2. 6; -14.                            | 16. 14; -1.                         | 30. $\frac{2}{3}$ ; $-\frac{8}{3}$ .  |
| 3. 9; -19.                            | 17. 12; -1.                         | 31. $\frac{2}{7}$ ; $-\frac{5}{7}$ .  |
| 4. 4; 3.                              | 18. 6; -10.                         | 32. $\frac{2}{2}$ ; $-\frac{1}{10}$ . |
| 5. 7; -2.                             | 19. $\frac{11}{2}$ ; -4.            | 33. $\frac{5}{6}$ ; $-\frac{14}{6}$ . |
| 6. 12; -5.                            | 20. 1; $-\frac{2}{3}$ .             | 34. 2; $-\frac{1}{5}$ .               |
| 7. 20; -19.                           | 21. 3; $-4\frac{2}{3}$ .            | 35. 2; -3.                            |
| 8. 5; -2.                             | 22. $\frac{1}{3}$ ; -2.             | 36. 5; $\frac{2}{3}$ .                |
| 9. 30; -21.                           | 23. $\frac{6}{5}$ ; $\frac{1}{5}$ . | 37. 5; $-\frac{10}{8}$ .              |
| 10. 25; -10.                          | 24. 10; $-\frac{5}{8}$ .            | 38. $\frac{2}{3}$ ; $-\frac{1}{5}$ .  |
| 11. $\frac{1}{2}$ ; $-\frac{7}{2}$ .  | 25. 2; $-\frac{5}{7}$ .             | 39. 5; $-2\frac{1}{2}$ .              |
| 12. 3; 2.                             | 26. 3; $-\frac{2}{3}$ .             | 40. $\frac{7}{8}$ ; $-\frac{23}{8}$ . |
| 13. $14\frac{2}{3}$ ; $\frac{1}{3}$ . | 27. 5; $\frac{2}{3}$ .              | 41. 2; $\frac{2}{3}$ .                |
| 14. 4; -6.                            | 28. 4; $-\frac{5}{2}$ .             | 42. 6; $-\frac{1}{2}$ .               |

- |  |  |                              |
|--|--|------------------------------|
| 43. 9; 2.  | 64. $2.3; -\frac{4}{3}$ .                    | 85. 6; -17.                  |
| 44. $11; \frac{7}{2}$ .                              | 65. $3.1; 0.6$ .                             | 86. 2; -1.                   |
| 45. $1; -\frac{1}{18}$ .                             | 66. $1\frac{1}{2}; -\frac{5}{8}$ .           | 87. 5; 4.                    |
| 46. $\frac{3}{2}; -\frac{15}{2}$ .                   | 67. $4; -2\frac{1}{2}$ .                     | 88. 7; $-\frac{1}{8}$ .      |
| 47. $\frac{4}{5}; 3\frac{1}{5}$ .                    | 68. $5; 2\frac{1}{2}$ .                      | 89. 1; $-1\frac{1}{2}$ .     |
| 48. $\frac{1}{8}; \frac{5}{8}$ .                     | 69. $10; -0.7$ .                             | 90. 4; -5.                   |
| 49. $\frac{2}{3}; -2$ .                              | 70. $14; 5\frac{1}{5}$ .                     | 91. 13; $\frac{1}{2}$ .      |
| 50. $1; \frac{8}{5}$ .                               | 71. $20; -1\frac{1}{8}$ .                    | 92. 7; $4\frac{5}{8}$ .      |
| 51. $-2; 2\frac{4}{5}$ .                             | 72. 18; 15.8.                                | 93. 8; $-8\frac{3}{4}$ .     |
| 52. $\frac{4}{5}; -\frac{4}{3}$ .                    | 73. $11\frac{1}{2}; 11$ .                    | 94. $15; \frac{983}{1647}$ . |
| 53. $\frac{2}{3}; 0$ .                               | 74. $16; -\frac{41}{105}$ .                  | 95. $\frac{2}{3}; -3$ .      |
| 54. $4; -3\frac{1}{8}$ .                             | 75. $22; 5\frac{4}{7}$ .                     | 96. 5; $1\frac{1}{5}$ .      |
| 55. $3; -4\frac{5}{7}$ .                             | 76. 7; -1.                                   | 97. 8; -1.                   |
| 56. 2; $-\frac{2}{5}$ .                              | 77. 4; -2.                                   | 98. 5; $-1\frac{1}{4}$ .     |
| 57. $\frac{1}{5}; -\frac{1}{7}$ .                    | 78. 12; 11.                                  | 99. -7; -11.                 |
| 58. $\frac{18}{10}; 0.3$ .                           | 79. $3\frac{1}{2}; -\frac{1}{2}$ .           | 100. -11; -13.               |
| 59. 0.5; 0.1.  | 80. $2\frac{1}{2}; \frac{1}{2}$ .            | 101. 7; $3\frac{1}{5}$ .     |
| 60. 5; -7.   | 81. $\frac{2}{3}; -\frac{1}{2}\frac{2}{1}$ . | 102. 0; $-\frac{2}{5}$ .     |
| 61. 7; 1.  | 82. 6; $-\frac{1}{8}$ .                      | 103. 0; $-\frac{5}{2}$ .     |
| 62. 13; $-\frac{13}{8}$ .                            | 83. 11; 1.                                   | 104. 0; $-\frac{243}{108}$ . |
| 63. 2; 40.   | 84. 17; 6.                                   | 105. 0; 1.                   |
| 106. $-\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2 - 4b}$ . | 111. $-\frac{1}{2}(m \pm \sqrt{m^2 - 4n})$ . |                              |
| 107. $\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2 - 4b}$ .  | 112. $\frac{1}{2}(d \pm \sqrt{d^2 + 4g})$ .  |                              |
| 108. $-\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2 + 4b}$ . | 113. $\frac{1}{2}(b \pm \sqrt{b^2 + 4c})$ .  |                              |
| 109. $\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2 + 4b}$ .  | 114. $4p; -p$ .                              |                              |
| 110. $\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2 + 4b}$ .  | 115. $2b; -b$ .                              |                              |
|  | 116. $-\frac{d}{2} \pm 2k^2$ .               |                              |

117.  $\frac{3}{2a}$ ;  $-\frac{1}{2a}$ .  
 118.  $-m$ ;  $-n$ .  
 119.  $n$ ;  $p$ .  
 120.  $\frac{m}{n}$ ;  $\frac{n}{m}$ .  
 121.  $\frac{3n}{4}$ ;  $-2\frac{1}{4}n$ .  
 122.  $c$ ;  $2$ .  
 123.  $a$ ;  $-\frac{a}{2}$ .  
 124.  $\frac{2m}{3}$ ;  $-m$ .  
 125.  $\frac{p}{2}$ ;  $\frac{p}{10}$ .  
 126.  $\frac{3b}{2}$ ;  $\frac{b}{6}$ .  
 127.  $\frac{3(a+b)}{2}$ ;  $-\frac{a+b}{2}$ .  
 128.  $m$ ;  $-n$ .  
 129.  $a$ ;  $-b$ .  
 130.  $m+n$ ;  $2n$ .  
 131.  $-a$ ;  $-b$ .  
 132.  $-\frac{1}{6}(a \pm \sqrt{a^2 + 12b})$ .  
 133.  $2a$ ;  $-\frac{a}{2}$ .  
 134.  $\frac{1}{2b}(a \pm \sqrt{a^2 - 4bp})$ .  
 135.  $1$ ;  $-\frac{5a+c}{c}$ .  
 136.  $-\frac{a}{2}$ ;  $-\frac{9a}{2}$ .  
 137.  $\frac{a}{2}$ ;  $\frac{b}{2}$ .  
 138.  $2p$ ;  $\frac{p}{2}$ .  
 139.  $\frac{ab}{2a-b}$ ;  $2b$ .  
 140.  $4a-3b$ ;  $3b-a$ .  
 141.  $\frac{a+3b}{2}$ ;  $-\frac{3a+b}{2}$ .  
 142.  $\frac{3p}{4}$ ;  $-p$ .  
 143.  $\frac{b}{a}$ ;  $-\frac{d}{c}$ .  
 144.  $\frac{2a-b}{ac}$ ;  $-\frac{3a+2b}{bc}$ .  
 145.  $\frac{a}{a-b}$ ;  $\frac{b}{a-b}$ .  
 146.  $\frac{1}{ab^2}$ ;  $\frac{1}{a^2b}$ .  
 147.  $-\frac{3a}{2}$ ;  $-4a$ .  
 148.  $\frac{b}{a} \pm \frac{1}{a}\sqrt{b^2 + ac}$ .  
 149.  $-\frac{b}{a} \pm \frac{1}{a}\sqrt{b^2 - ac}$ .  
 150.  $\frac{2q}{p} \pm \frac{1}{p}\sqrt{4q^2 + mp}$ .  
 151.  $-2 \pm \frac{1}{m}\sqrt{4m^2 - cm}$ .  
 152.  $\frac{8+a}{c}$ ;  $-\frac{a}{c}$ .  
 153.  $5a$ ;  $\frac{7a}{3}$ .  
 154.  $\frac{7ab}{3}$ ;  $\frac{3ab}{5}$ .

155.  $ab \pm a\sqrt{b^2 - b}$ .  
 156.  $\frac{2m}{3}; -\frac{2n}{3}$ .  
 157.  $\frac{a+b-ab}{b+1}; -\frac{a+b+ab}{b+1}$ .  
 158.  $-\frac{a-b}{3}; -(a-b)$ .  
 159.  $a+b; 3b-a$ .  
 160.  $\frac{b^2}{ac}$ .  
 161.  $a+2b; a-2b$ .  
 162.  $a^2+b^2 \pm \sqrt{a^4+b^4}$ .  
 163.  $-c \pm \sqrt{c^2+ab-bc-ac}$ .  
 164.  $-\frac{a}{2}(3 \pm \sqrt{3})$ .  
 165.  $\frac{a}{2}(3 \pm \sqrt{3})$ .  
 166.  $\frac{2a}{3}; 0$ .  
 167.  $-\frac{b+2a}{ab} \pm \frac{3}{ab}\sqrt{ab}$ .  
 168.  $\frac{ab}{c}; -\frac{c^2}{ab}$ .  
 169.  $\frac{a}{b}; -\frac{b}{a}$ .  
 170.  $3a-2b; 2a+3b$ .  
 171.  $2b+a; 2b-a$ .  
 172.  $a+2c; b-c$ .  
 173.  $a+b-2c; b+2c$ .  
 174.  $a^2+b^2; 2ab$ .  
 175.  $\frac{3b}{c}; -\frac{5d}{a}$ .  
 176.  $\frac{a^3b}{c^4}; -\frac{b^2c^3}{a^5}$ .  
 177.  $\frac{2n^4}{m^6p^2}; -\frac{2n^2}{m^3p^5}$ .  
 178.  $a^2-b^2; a+b$ .  
 179.  $2m(2m-n); n(2m+n)$ .  
 180.  $\frac{2ab-ac-bc}{a+b-2c}; 0$ .  
 181.  $c; -\frac{d(c+d)}{2c+d}$ .  
 182.  $2p; 0$ .  
 183.  $0; 13a-4b+d$ .  
 184.  $0; \frac{bp-cn}{ap-cm}$ .

## Ex. 28. Page 107.

1.  $\pm 35; \pm 21$ .  
 2.  $\pm 19; \pm 20; \pm 21$ .  
 3.  $\pm 14; \pm 16; \pm 18$ .  
 4.  $\pm 28; \pm 32; \pm 36$ .  
 5. \$229,376.  
 6.  $\pm 59; \pm 47$ .  
 7.  $(12\sqrt{3})^m; (6\sqrt{3})^m$ .  
 8. 96; 6.  
 9. 5 dys.  
 10. 3<sup>rs</sup>.  
 11. 5.996 dys.

$$45. \begin{aligned} x &= 5(c+d), \\ y &= 3(c-d). \end{aligned}$$

$$50. \begin{aligned} x &= \frac{c-d}{c+d}, \\ y &= \frac{c+d}{c-d}. \end{aligned}$$

$$46. \begin{aligned} x &= a(a+1) + b(b-1), \\ y &= a(a-1) - b(b-1). \end{aligned}$$

$$51. \begin{aligned} x &= \frac{m+n^2}{2n}, \\ y &= \frac{m-n^2}{2n}. \end{aligned}$$

$$47. \begin{aligned} x &= a - 2b + 3c, \\ y &= 3a - 2b + c. \end{aligned}$$

$$52. \begin{aligned} x &= \frac{m^2+n^2}{2n}, \\ y &= \frac{m^2-n^2}{2n}. \end{aligned}$$

$$48. \begin{aligned} x &= m + 3n + 2p, \\ y &= 3m - n + 2p. \end{aligned}$$

$$49. \begin{aligned} x &= \frac{m^2-n^2}{m}, \\ y &= \frac{m^2-n^2}{n}. \end{aligned}$$

$$53. \begin{aligned} x &= c^3 - d^3, \\ y &= c^3 + d^3. \end{aligned}$$

$$54. \begin{aligned} x &= a^5 + b^5, \\ y &= a^5 - b^5. \end{aligned}$$

## Ex. 22. Page 80.

$$1. \begin{aligned} x &= 2, \\ y &= 3, \\ z &= 4. \end{aligned}$$

$$6. \begin{aligned} x &= 10, \\ y &= 20, \\ z &= 30. \end{aligned}$$

$$11. \begin{aligned} x &= 3, \\ y &= 2, \\ z &= 1. \end{aligned}$$

$$16. \begin{aligned} x &= 4, \\ y &= 6, \\ z &= 8. \end{aligned}$$

$$2. \begin{aligned} x &= 1, \\ y &= 3, \\ z &= 5. \end{aligned}$$

$$7. \begin{aligned} x &= 1.7, \\ y &= 1.5, \\ z &= 1.3. \end{aligned}$$

$$12. \begin{aligned} x &= 6, \\ y &= 8, \\ z &= 10. \end{aligned}$$

$$17. \begin{aligned} x &= 8, \\ y &= 6, \\ z &= 2. \end{aligned}$$

$$3. \begin{aligned} x &= 3, \\ y &= 5, \\ z &= 7. \end{aligned}$$

$$8. \begin{aligned} x &= 11, \\ y &= 7, \\ z &= 9. \end{aligned}$$

$$13. \begin{aligned} x &= 11, \\ y &= 12, \\ z &= 13. \end{aligned}$$

$$18. \begin{aligned} x &= 1\frac{2}{3}, \\ y &= 4\frac{5}{8}, \\ z &= 7\frac{3}{8}. \end{aligned}$$

$$4. \begin{aligned} x &= 1, \\ y &= 2, \\ z &= 3. \end{aligned}$$

$$9. \begin{aligned} x &= 12, \\ y &= 18, \\ z &= 35. \end{aligned}$$

$$14. \begin{aligned} x &= 12, \\ y &= 34, \\ z &= 56. \end{aligned}$$

$$19. \begin{aligned} x &= -1.2, \\ y &= 3.4, \\ z &= -5.6. \end{aligned}$$

$$5. \begin{aligned} x &= 2, \\ y &= 4, \\ z &= 8. \end{aligned}$$

$$10. \begin{aligned} x &= 15, \\ y &= 6, \\ z &= 8. \end{aligned}$$

$$15. \begin{aligned} x &= 50, \\ y &= 31, \\ z &= 19. \end{aligned}$$

$$20. \begin{aligned} x &= 20, \\ y &= 10, \\ z &= 5. \end{aligned}$$

- |   |  |  |  |
|---|--|--|--|
| 21. $x = 15,$<br>$y = 12,$<br>$z = 10.$                     | 32. $x = 10,$<br>$y = 2,$<br>$z = 3.$      | 43. $x = 3,$<br>$y = 4,$<br>$z = 5.$       | 54. $x = 3,$<br>$y = 2,$<br>$z = 4.$   |
| 22. $x = 1,$<br>$y = 2,$<br>$z = 3.$                        | 33. $x = 4,$<br>$y = 3,$<br>$z = 5.$       | 44. $x = 10,$<br>$y = 7,$<br>$z = 3.$      | 55. $x = \frac{4}{3},$<br>$y = 4,$<br>$z = \frac{4}{3}.$                       |
| 23. $x = \frac{3}{8},$<br>$y = -7,$<br>$z = 36\frac{1}{8}.$ | 34. $x = 3,$<br>$y = 7,$<br>$z = 4.$       | 45. $x = 51,$<br>$y = 76,$<br>$z = 1.$     | 56. $x = 1\frac{1}{2},$<br>$y = -3\frac{1}{2},$<br>$z = 2\frac{1}{16}.$        |
| 24. $x = 7,$<br>$y = 8,$<br>$z = 9.$                        | 35. $x = 13,$<br>$y = 24,$<br>$z = 62.$    | 46. $x = 9,$<br>$y = 10,$<br>$z = 11.$     | 57. $x = \frac{1}{2},$<br>$y = \frac{3}{8},$<br>$z = \frac{3}{4}.$             |
| 25. $x = 13,$<br>$y = 65,$<br>$z = 91.$                     | 36. $x = 1,$<br>$y = 2,$<br>$z = 3.$       | 47. $x = 2,$<br>$y = 3,$<br>$z = 4.$       | 58. $x = \frac{1}{2},$<br>$y = \frac{1}{8},$<br>$z = \frac{1}{4}.$             |
| 26. $x = 12,$<br>$y = 18,$<br>$z = 24.$                     | 37. $x = 10,$<br>$y = 20,$<br>$z = 30.$    | 48. $x = 0,$<br>$y = 1,$<br>$z = 3.$       | 59. $x = \frac{1}{2},$<br>$y = \frac{1}{8},$<br>$z = \frac{1}{4}.$             |
| 27. $x = 3,$<br>$y = 4,$<br>$z = 5.$                        | 38. $x = 1,$<br>$y = 2,$<br>$z = 3.$       | 49. $x = 1,$<br>$y = 2,$<br>$z = 3.$       | 60. $x = 5,$<br>$y = 4,$<br>$z = 3.$   |
| 28. $x = 1,$<br>$y = 2,$<br>$z = 3.$                        | 39. $x = 24,$<br>$y = 60,$<br>$z = 120.$   | 50. $x = 111,$<br>$y = 222,$<br>$z = 333.$ | 61. $x = 1,$<br>$y = 4,$<br>$z = \frac{1}{2},$<br>$u = 1\frac{1}{2}.$          |
| 29. $x = 5,$<br>$y = 6,$<br>$z = 8.$                        | 40. $x = 315,$<br>$y = 630,$<br>$z = 945.$ | 51. $x = 5,$<br>$y = 7,$<br>$z = 3.$       | 62. $x = 100,$<br>$y = 60,$<br>$z = -13,$<br>$u = -50.$                        |
| 30. $x = 1,$<br>$y = 4,$<br>$z = 6.$                        | 41. $x = 2,$<br>$y = 2,$<br>$z = 2.$       | 52. $x = 11,$<br>$y = 3,$<br>$z = 7.$      | 63. $x = \frac{1}{2},$<br>$y = \frac{1}{2},$<br>$z = \frac{1}{2},$<br>$u = 0.$ |
| 31. $x = 2,$<br>$y = 9,$<br>$z = 10.$                       | 42. $x = 0,$<br>$y = 1,$<br>$z = 2.$       | 53. $x = 3,$<br>$y = 7,$<br>$z = 11.$      | 64. $x = 12,$<br>$y = 5,$<br>$z = 7,$<br>$u = 4.$                              |

$$65. x = 3, y = 1, z = 5, u = 9.$$

$$66. x = 12, y = 30, z = 168, u = 50.$$

## Ex. 23. Page 86.

$$\begin{aligned} 1. \quad x &= \frac{1}{2}(a + b - c), \\ y &= \frac{1}{2}(a + c - b), \\ z &= \frac{1}{2}(b + c - a). \end{aligned}$$

$$\begin{aligned} 2. \quad x &= \frac{m + p - t}{m - q}, \\ y &= \frac{mt - mp - nq}{m - q}, \\ z &= \frac{mt - nq - pq}{m - q}. \end{aligned}$$

$$\begin{aligned} 3. \quad x &= \frac{n + p}{2}, \\ y &= \frac{m - n}{2}, \\ z &= \frac{m - p}{2}. \end{aligned}$$

$$\begin{aligned} 4. \quad x &= -abc, \\ y &= ac + ab + bc, \\ z &= -(a + b + c). \end{aligned}$$

$$\begin{aligned} 5. \quad x &= b + c, \\ y &= a + c, \\ z &= a + b. \end{aligned}$$

$$\begin{aligned} 6. \quad x &= \frac{p}{2 - m - n + mn}, \\ y &= \frac{p(m - 1)}{2 - m - n + mn}, \\ z &= \frac{p(1 - m - n + mn)}{2 - m - n + mn}. \end{aligned}$$

$$\begin{aligned} 7. \quad x &= \frac{amp}{mp + np + nq}, \\ y &= \frac{anp}{mp + np + nq}, \\ z &= \frac{anq}{mp + np + nq}. \end{aligned}$$

$$\begin{aligned} 8. \quad x &= \frac{2}{m + n - p}, \\ y &= \frac{2}{m - n + p}, \\ z &= \frac{2}{n + p - m}. \end{aligned}$$

$$\begin{aligned} 9. \quad x &= \frac{2a}{m + n}, \\ y &= \frac{2b}{m + p}, \\ z &= \frac{2c}{n + p}. \end{aligned}$$

$$\begin{aligned} 10. \quad x &= a, \\ y &= b, \\ z &= c. \end{aligned}$$

$$\begin{aligned} 11. \quad x &= \frac{mp - mn + np}{2mnp}, \\ y &= \frac{mn + np - mp}{2mnp}, \\ z &= \frac{mn + mp - np}{2mnp}. \end{aligned}$$

$$12. \quad x = \frac{2abc}{ac + bc - ab},$$

$$y = \frac{2abc}{ab + bc - ac},$$

$$z = \frac{2abc}{ab + ac - bc}.$$

$$13. \quad x = \frac{abc}{ab + bc + ac},$$

$$y = \frac{abc}{ab + bc + ac},$$

$$z = \frac{abc}{ab + bc + ac}.$$

$$14. \quad x = m^2 - n^2,$$

$$y = n^2 - p^2,$$

$$z = p^2 - m^2.$$

$$15. \quad x = (m + n)(n - p),$$

$$y = (n + p)(p - m),$$

$$z = (p + m)(m - n).$$

$$16. \quad x = n^2 - p^2,$$

$$y = p^2 - m^2,$$

$$z = m^2 - n^2.$$

$$17. \quad x = \frac{1}{n - p},$$

$$y = \frac{1}{p - m},$$

$$z = \frac{1}{m - n}.$$

$$18. \quad x = n + p - m,$$

$$y = m + p - n,$$

$$z = m + n - p.$$

$$19. \quad x = \frac{1}{2}(n + p),$$

$$y = \frac{1}{2}(p + m),$$

$$z = \frac{1}{2}(m + n).$$

20. Placing

$$s = \frac{1}{2}(m + n + p + q)$$

we have

$$x = s - m,$$

$$y = s - n,$$

$$z = s - p,$$

$$u = s - q.$$

Putting

$$s = \frac{1}{2}(l + m + n + p + q)$$

we have

$$x = s - q,$$

$$y = s - p,$$

$$z = s,$$

$$u = s - m,$$

$$v = s - n,$$

$$w = s - l.$$

$$21. \quad x = p + n,$$

$$y = q - p,$$

$$z = m + p - q,$$

$$u = q + n - m,$$

$$v = m - n.$$

$$22. \quad x = m + n + p,$$

$$y = n - m - p,$$

$$z = 3m - n + p,$$

$$u = m + n + p.$$



42. Min. =  $20\frac{2}{3}$  for  $x = \frac{2}{3}$ .

Max. = 30 for  $x = -2$ .

43. Max. = 3.1219 for  $x = \frac{2 - \sqrt{7}}{3}$ .

Min. = 0.369 for  $x = \frac{2 + \sqrt{7}}{3}$ .

44. Max. =  $19\frac{1}{3}$  for  $x = -\frac{1}{3}$ .

Min. = 6 for  $x = 2$ .

45. Max. =  $-20\frac{1}{3}$  for  $x = 1$ .

Min. =  $-20\frac{1}{3}$  for  $x = 2$ .

46. When  $x$  increases from  $y$  varies from

$-\infty$  to  $+1$ ,

$+\infty$  to 0,

1 to 3,

0 to  $-4$  (min.),

3 to 5,

$-4$  to 0,

5 to  $+\infty$ ,

0 to  $+\infty$ .

47. When  $x$  varies from

$y$  varies from

$-\infty$  to  $-3$ ,

$+\infty$  to 0,

$-3$  to  $-2$ ,

0 to  $-25$  (min.),

$-2$  to 0,

$-25$  to  $-9$  (max.),

0 to 2,

$-9$  to  $-25$  (min.),

2 to 3,

$-25$  to 0,

3 to  $+\infty$ ,

0 to  $+\infty$ .

48. When  $x$  varies from

$y$  varies from

$-\infty$  to  $-1$ ,

$-\infty$  to  $-2$  (max.),

$-1$  to  $-h$ ,

$-2$  to  $-\infty$ ,

$-h$  to  $+h$ ,

$-\infty$  to  $+\infty$ ,

$+h$  to 1,

$+\infty$  to 2 (min.),

1 to  $\infty$ ,

2 to  $\infty$ .

49. When  $x$  varies from

$y$  varies from

$-\infty$  to 1,

$+\infty$  to  $-2$  (min.),

1 to 2,

$-2$  to  $+\infty$ ,

2 to 3,

$+\infty$  to  $-6$  (max.),

3 to  $\infty$ ,

$-6$  to  $-\infty$ .

50. When  $x$  varies from  $y$  varies from  
 $-\infty$  to  $-1$ ,  $1$  to  $\frac{7}{3}$  (max.),  
 $-1$  to  $0$ ,  $\frac{7}{3}$  to  $1$ ,  
 $0$  to  $+1$ ,  $1$  to  $-3$  (min.),  
 $+1$  to  $+\infty$ ,  $-3$  to  $1$ .
51. When  $x$  varies from  $y$  varies from  
 $-\infty$  to  $-7$ ,  $0$  to  $\frac{1}{14}$  (max.),  
 $-7$  to  $-3$ ,  $\frac{1}{14}$  to  $-\frac{1}{12}$  (min.),  
 $-3$  to  $-1$ ,  $-\frac{1}{12}$  to  $\frac{1}{2}$  (max.),  
 $-1$  to  $+2$ ,  $\frac{1}{2}$  to  $2$ ,  
 $2$  to  $5$ ,  $2$  to  $-\frac{1}{2}$ .
52. Let  $x$  be one part;  $2a-x$  = the other; then  $(2a-x)x = m$ ; whence  $x = a$ . The parts should be equal.
53. A square. 54. Each part =  $a$ . 55. Each part =  $a$ .
56. Let  $a, b, c$  be the sides of a triangle, and  $2p$  be its perimeter. The surface is  $\sqrt{p(p-a)(p-b)(p-c)} = m$ . Now  $p$  is constant; in order that there may be a maximum, it is necessary that  $p-a = p-b = p-c$ ; that is to say,  $a = b = c$ .
57. Let  $a$  and  $b$  be the sides of a rectangle;  $r$  the radius of the circle. The area of the triangle is  $ab = m$ . But  $a^2 + b^2 = 4r^2$ , or  $a^2 + \frac{m^2}{a^2} = 4r^2$ ; whence  $a^2 = 2r^2 \pm \sqrt{4r^2 - m^2}$ . The radical must be equal to zero, which makes  $a = r\sqrt{2}$ , and  $m = r\sqrt{2} \times r\sqrt{2} = 2r^2$ . The figure is then the inscribed square.
58. Let  $a$  be the base, and  $h$  the height of the triangle;  $x$  the base, and  $y$  the height of the rectangle. Then the surface  $xy = m$ . But  $\frac{h}{h-y} = \frac{a}{x}$ ; whence  $x = \frac{a(h-y)}{h}$ , and  $xy = \frac{(ah-ay)y}{h} = m$ ; or  $ay^2 - ahy = -mh$ ; whence  $y = \frac{h}{2} \pm \sqrt{\frac{h^2}{4} - \frac{mh}{a}}$ . If the radical = 0,  $y = \frac{h}{2}$ ,  $x = \frac{a}{2}$ .

59. Let  $x$  be the radius of the cylinder, and  $y$  its height.

Then  $\frac{h}{h-y} = \frac{r}{x}$ ; whence  $x = \frac{r(h-y)}{h}$ . The volume of the cylinder is  $\pi x^2 y = \frac{\pi r^2}{h^2} (h-y)^2 y$ . The maximum depends solely upon the factors  $(h-y)^2$  and  $y$ , the sum of whose first powers is constant. Then we must have for the product, as maximum,  $\frac{h-y}{2} = \frac{y}{1}$ ; whence  $y = \frac{h}{3}$ , and  $x = \frac{2r}{3}$ .

60. Start from each angle of the given square, and, going in the same direction, take the distances equal  $x$ , and join two and two the consecutive points thus obtained. The figure inscribed will be a square. Let  $h$  be the side of this square, of which the area  $h^2 = M$ . But  $h^2 = x^2 + (a-x)^2 = 2x^2 - 2ax + a^2 = M$ . Then  $x = \frac{a}{2} \pm \frac{1}{2} \sqrt{2M - a^2}$ . Then,  $M = \frac{a^2}{2}$ ,  $x = \frac{a}{2}$ , and  $h = \frac{a\sqrt{2}}{2}$ .

61. Let  $R$  be the radius of the base of the cone, and  $H$  be its height. The volume, to be a minimum, is  $\frac{\pi R^2 H}{3}$ . But  $\frac{H}{H-h} = \frac{R}{r}$ ; whence  $R = \frac{Hr}{H-h}$ ; then  $\frac{\pi R^2 H}{3} = \frac{\pi H^3 r^2}{3(H-h)^2} = \frac{\pi r^2}{3} \times \frac{H^3}{(H-h)^2}$ . The minimum depends only on  $\frac{H^3}{(H-h)^2}$ , which can be written

$$\frac{H^3}{H^2 - 2Hh + h^2} = \frac{1}{\frac{H}{h} \left(1 - \frac{h}{H}\right)^2} = \frac{h}{\frac{H}{h} \left(1 - \frac{h}{H}\right)^2}.$$

The first powers of the two factors of the denominator give a constant sum; their product is a maximum when we

have  $\frac{\frac{h}{H}}{1} = \frac{1 - \frac{h}{H}}{2}$ ; whence  $H = 3h$ , and  $R = \frac{3r}{2}$ .

62. Let  $R$  be the radius of the sphere;  $r$  the radius of the base of the cylinder;  $h$  its height. Then  $\frac{h^2}{4} + r^2 = R^2$ ; then  $r^2 = R^2 - \frac{h^2}{4}$ . The surface of the cylinder  $= 2\pi rh$ ; but  $2\pi$  is constant, then  $rh = \text{max.} = m$ ; whence  $r^2 = \frac{m^2}{h^2}$ , and  $R^2 - \frac{h^2}{4} = \frac{m^2}{h^2}$ . Therefore,  $h^2 = 2R^2 \pm 2\sqrt{R^4 - m^2}$ . Then  $R^4$  must  $= m^2$ ; then  $m = R^2$ ;  $h = R\sqrt{2}$ , and  $r = \frac{R\sqrt{2}}{2}$ . The height and the diameter are equal to a side of the square inscribed.

63. Let  $2x$  be the base of the triangle;  $h$  its height;  $R$  the radius of the circle circumscribed. Surface  $xh = m$ . But  $x^2 + (h - R)^2 = R^2$ ; then  $x = \sqrt{2hR - h^2}$ . Then  $h^2(2hR - h^2) = m^2 = h^3(2R - h)$ . The sum  $2R - h + h$  being constant, we have  $\frac{h}{3} = \frac{2R - h}{1}$ ; whence  $h = \frac{3R}{2}$ , and  $x = \frac{R\sqrt{3}}{2}$ ; the base is then  $R\sqrt{3} = \text{side of the equilateral triangle.}$

64. Let  $R$  be the radius of the sphere;  $r$  the radius of the base of the cone;  $h$  the height of the cone;  $D$  the distance from the summit to the circle of contact with the sphere. We have  $\frac{r}{R} = \frac{h}{D}$ . But  $D^2 = (h - R)^2 - R^2$ ; then  $\frac{r}{R} = \frac{h}{\sqrt{(h - R)^2 - R^2}}$ ; whence  $r^2 = \frac{R^2 h}{h - 2R}$ . The volume of the cone is  $\frac{\pi r^2 h}{3} = \frac{\pi}{3} \times \frac{R^2 h^2}{h - 2R} = m$ ; as  $\frac{\pi}{3}$  is constant it is suppressed, and

$$h = \frac{m}{2R^2} \pm \sqrt{\frac{m^2}{4R^4} - \frac{8mR^3}{4R^4}}.$$

Making the radical equal to zero, we have  $m = 8R^3$ ;  $h = 4R$ ; and  $r = R\sqrt{2}$ .

65. Let  $R$  be the radius of the sphere;  $r$  the radius of the base of the cone;  $h$  its height;  $A$  its apothem. The surface, to render maximum, is  $\pi r A$ , or  $r A$ ,  $\pi$  being constant. Now  $r^2 = (R+x)(R-x)$ ;  $A^2 = 2R(R+x)$ ; whence  $r^2 A^2 = 2R(R+x)^2(R-x)$ . Now  $2R$  is constant, and the sum of the factors  $R+x$  and  $R-x$  is constant; then  $\frac{R+x}{2} = R-x$ ; then  $x = \frac{R}{3}$ , and  $h = \frac{4R}{3}$ .

## Ex. 31. Page 119.

- |   |  |  |
|---|--|--|
| 1. $x = \pm 25$ ;<br>$y = \pm 6$ .  | 9. $x = 7, 5$ ;<br>$y = 5, 7$ .                          | 16. $x = \frac{3}{4}, \frac{1}{4}$ ;<br>$y = \frac{1}{8}, \frac{3}{8}$ . |
| 2. $y = \pm 1$ ;<br>$z = \pm 9$ .   | 10. $x = 13, -13$ ;<br>$y = 11, -11$ .                   | 17. $x = 13, 9$ ;<br>$y = -9, -13$ .                                     |
| 3. $x = \pm 5$ ;<br>$y = \pm 4$ .   | 11. $x = 9, -12$ ;<br>$y = 12, -9$ .                     | 18. $x = 13$ ;<br>$y = 7$ .  |
| 4. $x = \pm 8$ ;<br>$y = \pm 6$ .   | 12. $x = 7, -3\frac{1}{3}$ ;<br>$y = 8, 12\frac{2}{3}$ . | 19. $x = \frac{3}{8}$ ;<br>$y = \frac{5}{8}$ .                           |
| 5. $x = 7, 5$ ;<br>$y = 5, 7$ .   | 13. $x = 3, -\frac{3}{2}$ ;<br>$y = 4, -\frac{11}{2}$ .  | 20. $x = 13, -13$ ;<br>$y = 7, -7$ .                                     |
| 6. $y = 20, -12$ ;<br>$z = 12, -20$ .   | 14. $x = 5, 2, 6, 1$ ;<br>$y = 2, 5, 1, 6$ .             | 21. $x = \pm 13$ ,<br>$\pm 10\sqrt{2}$ ;                                 |
| 7. $x = \frac{3}{8}, \frac{1}{8}$ ;<br>$y = \frac{1}{2}, \frac{3}{8}$ .                                     | 15. $y = 11, 3$ ;<br>$z = 3, 11$ .                       | $y = \pm 7$ ,<br>$\pm 3\sqrt{2}$ .                                       |
| 8. $x = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$ ;<br>$y = \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}$ . | 22. $x = 7, -\frac{5}{2}$ ;<br>$y = -3, 17\frac{1}{2}$ . | 23. $x = \pm 3$ ;<br>$y = \pm 1$ .                                       |

24.  $x = 5, 14\frac{5}{11}$ ;  $y = 2, 15\frac{4}{11}$ . 29.  $x = 8, 13$ ;  $y = 12, 7$ . 34.  $x = \pm 7, \pm 3$ ;  $y = \pm 3, \pm 7$ .
25.  $x = 7, 5$ ;  $y = 5, 7$ . 30.  $x = 7, 4\frac{2}{11}$ ;  $y = 2, 3\frac{5}{11}$ . 35.  $x = \pm 11, \pm 6$ ;  $y = \pm 6, \pm 11$ .
26.  $x = 9, 8$ ;  $y = 8, 9$ . 31.  $x = 4, 5$ ;  $y = 5, 4$ . 36.  $x = \pm 14, \pm 12$ ;  $y = \pm 12, \pm 14$ .
27.  $x = 12$ ;  $y = 11$ . 32.  $x = 6, 3$ ;  $y = 3, 6$ . 37.  $x = 7, 2$ ;  $y = 2, 7$ .
28.  $x = 5, 4$ ;  $y = 3, 4$ . 33.  $x = 5, -12$ ;  $y = 12, -5$ . 38.  $x = 4, \frac{1}{2}$ ;  $y = \frac{1}{2}, 4$ .
39.  $x = 7, -\frac{3}{8}$ ;  $y = \frac{3}{8}, -7$ . 42.  $x = \frac{3}{4}, -\frac{7}{4}, \frac{3}{4}, -\frac{7}{4}$ ;  $y = \frac{1}{4}, -\frac{5}{4}, -\frac{5}{4}, \frac{1}{4}$ .
40.  $x = 10, -8$ ;  $y = 8, -10$ . 43.  $x = 3, 2, -3 \pm \sqrt{3}$ ;  $y = 2, 3, -3 \pm \sqrt{3}$ .
41.  $x = 5, -6, 5, -6$ ;  $y = 11, -12, -12, 11$ . 44.  $x = 4, 1$ ;  $y = 1, 4$ .
45.  $x = 11, -3$ ;  $y = 3, -11$ . 51.  $x = 7, -2$ ;  $y = 2, -7$ . 57.  $x = 8, 7$ ;  $y = 7, 8$ .
46.  $x = 5, 3$ ;  $y = 3, 5$ . 52.  $x = 8, 7$ ;  $y = 7, 8$ . 58.  $x = 4, 3$ ;  $y = 3, 4$ .
47.  $x = 7, -4$ ;  $y = 4, -7$ . 53.  $x = 11, -9$ ;  $y = 9, -11$ . 59.  $x = 3$ ;  $y = 1$ .
48.  $x = 6, 4$ ;  $y = 4, 6$ . 54.  $y = 1$ ;  $x = 4$ . 60.  $x = 6, 4\frac{1}{4}$ ;  $y = 4, \frac{39}{4}$ .
49.  $x = \pm 2, \pm \frac{58}{9}$ ;  $y = \pm 3, \pm \frac{47}{9}$ . 55.  $x = 6, 5$ ;  $y = 5, 6$ . 61.  $x = 6, -2$ ;  $y = -1, 6$ .
50.  $x = 5, 3$ ;  $y = 3, 5$ . 56.  $x = 5, 3$ ;  $y = 3, 5$ .

62.  $x = 12$ ;  
 $y = 6$ ;  
 $z = 9$ .
63.  $x = \pm 9$ ;  
 $y = \pm 7$ ;  
 $z = \pm 3$ .
64.  $x = 8, -16$ ;  
 $y = 4, -2$ ;  
 $z = 6, \frac{4}{3}$ .
65.  $x = \pm 6$ ;  
 $y = \pm 8$ ;  
 $z = \pm \frac{1}{2}$ .
66.  $x = \frac{8}{3}, \frac{3}{2}$ ;  
 $y = \frac{3}{2}, \frac{8}{3}$ ;  
 $z = 2, 2$ .
67.  $x = 20, -20$ ;  
 $y = 4, -4$ .
68.  $x = 11, -20$ ;  
 $y = 8, -\frac{22}{5}$ .
69.  $x = 7, -22$ ;  
 $y = 8, 37$ .
70.  $x = 2, -7$ ;  
 $y = 5, 4$ ;  
 $z = 4, 5$ ;  
 $u = 3, 12$ .
71.  $x = 5, \frac{59}{17}$ ;  
 $y = 1, \frac{52}{17}$ ;  
 $z = 6, \frac{123}{17}$ ;  
 $u = 4, \frac{47}{17}$ .

## Ex. 32. Page 122.

1. 128 sheep; \$2.50 per head.
2. 50 sheep; \$6.40 per head.
3. 42 cabbages, at \$1.20 a doz.; or 60 cabbages, at \$0.84 a doz.
4. 48 melons, at  $12\frac{1}{2}$  cents apiece; or 40 melons, at 15 cents apiece.
5. Blue, 60 yds., at \$6, or  $24\frac{6}{11}$  yds., at  $\$14\frac{2}{11}$ ; black, 30 yds., at \$16, or  $65\frac{5}{11}$  yds., at  $\$7\frac{1}{11}$ .
6. Front wheel,  $2.5^m$ ; back wheel,  $3.9^m$ .
7. 1200 men; ration,  $1^{ks}$ .
8. 1800 men; ration,  $2\frac{1}{2}$  lbs.
9. 28 workmen and  $24^{ks}$ ; or 36 workmen and  $56^{ks}$ .
10. Carriage, 54 hours; train, 27 hours.
11.  $150^{km}$ .
12.  $x = 80^m$ ;  $y = 39^m$ .

13. Length, 63 ft. ; width, 16 ft.  
 14. The greater, 28 ft. ; the smaller, 21 ft.  
 15. 36 yds. and 29 yds.      16. 12 ft. and 36 ft.  
 17. Watch, \$18; cane, \$6; umbrella, \$12; ring, \$9.  
 18. A, \$58; B, \$44; C, \$62; D, \$34.      19. 8473.  
 20.  $x = \frac{b+1}{2} \sqrt{\frac{a}{b}}$ ;  $y = \frac{b-1}{2} \sqrt{\frac{a}{b}}$ .

**Ex. 33. Page 126.**

- |   |  |
|---|--|
| 1. $x = 11, 10, \dots$ ;<br>$y = 1, 2, \dots$                   | 11. $x = 116, 96, \dots, 16$ ;<br>$y = 1, 4, \dots, 16$ .        |
| 2. $x = 1, 3, 5, 7$ ;<br>$y = 36, 25, 14, 3$ .                  | 12. $x = 35, 34, 33, \dots, 1$ ;<br>$y = 3, 6, 9, \dots, 105$ .  |
| 3. $x = 9$ ;<br>$y = 3$ .                                       | 13. $x = 18, 15, 12, \dots, 3$ ;<br>$y = 5, 10, 15, \dots, 30$ . |
| 4. $x = 2$ ;<br>$y = 1$ .                                       | 14. $x = 22, 21, 20, \dots, 1$ ;<br>$y = 1, 3, 5, \dots, 43$ .   |
| 5. $x = 9, 1$ ;<br>$y = 3, 6$ .                                 | 15. $x = 5, 10, 15$ ;<br>$y = 21, 14, 7$ .                       |
| 6. $x = 50, 37, 24, 11$ ;<br>$y = 1, 9, 17, 25$ .               | 16. $x = 13$ ;<br>$y = 8$ .                                      |
| 7. $x = 1, 4, 7, \dots$ ;<br>$y = 142, 137, 132, \dots$         | 17. $x = 2$ ;<br>$y = 10$ .                                      |
| 8. $x = 54, 45, 36, \dots, 9$ ;<br>$y = 8, 16, 24, \dots, 48$ . | 18. $x = 17, 10, 3$ ;<br>$y = 4, 8, 12$ .                        |
| 9. $x = 12, 3$ ;<br>$y = 3, 9$ .                                | 19. $x = 22, 69, 116, \dots$ ;<br>$y = 7, 22, 37, \dots$         |
| 10. $x = 11, 2$ ;<br>$y = 2, 5$ .                               | 20. $x = 16, 39, 62, \dots$ ;<br>$y = 65, 159, 253, \dots$       |



21.  $x = 150, 133, 116, 99, 82, 65, 48, 31, 14$ ;  
 $y = 5, 12, 19, 26, 33, 40, 47, 54, 61$ .
22.  $x = 14, 31, 48, 65, 82$ ;      26.  $x = 264, 121$ ;  
 $y = 6, 14, 22, 30, 38$ .       $y = 52, 195$ .
23.  $x = 6, 21, 36, 51, 66$ ;      27.  $x = 63$ ;  
 $y = 2, 9, 16, 23, 30$ .       $y = 55$ .
24.  $x = 4, 24, 44, 64, \dots$ ;      28. 39, 275.  
 $y = 10, 30, 50, 70, \dots$
25.  $x = 81, 57, 33, 9$ ;      29. 15, 71, 127, 183, 239, ....  
 $y = 8, 32, 56, 80$ .
30. First number, 213, 301, 389, ....;  
 second number, 62, 150, 238, ....
31. First part, 303, 240, 177, 114, 51;  
 second part, 49, 112, 175, 238, 301.
32.  $x = 16, 13, 10, 7, 4, 1$ ;      33.  $x = 8, 19, 30, 41, \dots$ ;  
 $y = 2, 4, 6, 8, 10, 12$ .       $y = 2, 15, 28, 41, \dots$
34. \$5 bills, 1, 3, 5, 7, 9, 11, 13, 15, 17;  
 \$2 bills, 41, 36, 31, 26, 21, 16, 11, 6, 1.
35. Calves, 3, 6, 9, 12, 15, 18, 21;  
 pigs, 33, 28, 23, 18, 13, 8, 3.
36. Lambs, 3, 10; geese, 39, 10.
37. Five-franc pieces, 9; one-franc pieces, 29.
38. Sovereigns, 20, 40, 60, ....;  
 twenty-cent pieces, 936, 3457, 5978, ....
39. Weights of 30<sup>g</sup>, 1, 2, 3, 4, 5, ....;  
 weights of 2½<sup>g</sup>, 388, 376, 364, 352, 340, ....
40. 11 grown persons; 6 children.
41. 334 pears.
42. 41, 34, 27, 20, 13, 6 yds. of silk;  
 1, 6, 11, 16, 21, 26 yds. of velvet.

43. First, \$5; second, \$4.  
 44. 41 peaches; 8 doz. apples.  
 45. 22 yrs. (born in 1858).  
 46. Born in 1786, and 22 yrs. old.  
 47. Potatoes, 1, 1, 2, 3, 4, 5, 6, 8;  
 apples, 2, 6, 3, 4, 1, 2, 3, 1;  
 squashes, 30, 5, 20, 10, 25, 15, 5, 10.  
 48. The number can be expressed in three ways:  $2x + 1$ ;  
 $3y + 2$ ;  $7z + 5$ ; whence  $2x + 1 = 3y + 2 = 7z + 5$ ,  
 which gives  $x = 23, 44, 65, 86, \dots$ ;  
 $y = 15, 29, 43, 57, \dots$ ;  
 $z = 6, 12, 18, 24, \dots$   
 The simultaneous values of the three unknown quantities give the same value for the numbers 47, 89, 131, 173, 215, .....
49. 503, 1007, 1511, 2015, .....  
 50. Calves, \$7; lambs, \$4; pigs, \$3.  
 51. 10, 45, 56.

**Ex. 34. Page 129.**

- |                                       |                                 |
|---------------------------------------|---------------------------------|
| 1. $13 > 5$ .                         | 11. $-8a < 2x$ .                |
| 2. $2a - 4 < x^2 + 2x$ .              | 12. $-(a - b)^2 < -(x + y)^2$ . |
| 3. $x^2 + y^2 + y - 2a > 7 - a + b$ . | 13. $18a^2 - a^2y^2 < 5a^2$ .   |
| 4. $x + 4y < 13$ .                    | 14. $3 > 2$ .                   |
| 5. $12 > 5$ .                         | 15. $21 > 13$ .                 |
| 6. $4 < 9$ .                          | 16. $a + \sqrt{b} > 1$ .        |
| 7. $a^2 - 2x < 3 - x - b^2$ .         | 17. $-(x^2 + 2b) < -7x^2$ .     |
| 8. $ax^2 > b^2 - 8$ .                 | 18. $a^2 - 1 < 70$ .            |
| 9. $21 > -7$ .                        | 19. $a^2 - b^2 > mn$ .          |
| 10. $-6 < -1$ .                       | 20. $a^2 - b^2 > x^2 - y^2$ .   |

21.  $5\frac{1}{2} > \frac{7}{2}$ .  
 22.  $x - y > \frac{1}{2}a$ .  
 23.  $900 > 144$ .  
 24.  $(a + b)^3 > (a - x)^3$ .  
 25.  $(x + 1)^4 < y^4$ .  
 26.  $a^5 < (b - 1)^5$ .  
 27.  $-1 > -32$ .  
 28.  $(3 - e)^7 > -1$ .  
 29.  $(1 + a)^4 > b^4$ .  
 30.  $-a^3(1 + m)^3 > x^3$ .  
 31.  $3 > 2$ .  
 32.  $-5 < +4$ .  
 33.  $9 > 7$ .  
 34.  $6 > 5$ , or  $-6 < -5$ .  
 48.  $x < -\frac{2}{5}$ ;  $y > \frac{3x}{2}$ ,  $< \frac{2x - 1}{3}$ .  
 49.  $x < -13$ ;  $y > \frac{10x + 1}{3}$ ,  $< 3x - 4$ .  
 50.  $x < -\frac{7}{6}$ ;  $y > 7x + 4$ ,  $< x - 3$ .  
 51.  $x > \frac{16}{3}$ ;  $y < -(x + 4)$ ,  $> \frac{8 - 5x}{2}$ .  
 52. Always.  
 53. Always.  
 54. Always.  
 55.  $x > 2$ , or  $< -3$ .  
 56.  $x$  included between  $-\frac{1}{2}$  and  $-\frac{3}{4}$ .  
 61.  $x$  included between 7 and  $-3$ .  
 62.  $x$  included between 6 and  $-2$ .  
 63.  $x$  included between  $\frac{5}{2}$  and  $-3$ .  
 64.  $x < \frac{1}{3}$ , or  $> \frac{5}{2}$ .  
 35.  $4 + ab > 2x - (a^2 + b^2)$ .  
 36.  $-(2x^2 + 1) < x$ .  
 37.  $x < y$ .  
 38.  $x > -\frac{1}{2}$ .  
 39.  $x > 6$ .  
 40.  $x > \frac{3}{2}$ .  
 41.  $x < 3\frac{1}{2}$ .  
 42.  $x < \frac{1}{2}$ .  
 43.  $x > -\frac{8m}{7}$ .  
 44.  $x > \frac{7}{8}$ .  
 45.  $x > \frac{3}{4}$ .  
 46.  $x < -2\frac{1}{2}$ .  
 47.  $x = 3$ ; then  $y < \frac{3}{4}$  and  $> 0$ .  
 57.  $x > -5$  and  $< 3$ .  
 58. Impossible.  
 59. Impossible.  
 60.  $x < 4$  and  $> -20$ .  
 65.  $x > \frac{7}{3}$ , or  $< \frac{1}{3}$ .  
 66.  $x > \frac{1}{4}$ , or  $< \frac{1}{5}$ .

## Ex. 35. Page 133.

- |   |   |   |
|---|---|---|
| 1. 192.   | 4. $d^{a+1}$ .                                  | 7. $(a+b)^{m+n}$ .  |
| 2. 1.844.   | 5. $a^{5x-2y}$ .                                | 8. $(x-y)^m$ .  |
| 3. $b^{m-1}$ .  | 6. $p^{a+6b}$ .                                 | 9. $m^{2x}-2m^x n^y + n^{2y}$ .                                 |
| 10. $p^{2(m-1)} + 2p^{m-1} q^{n-2} + q^{2(n-2)}$ .                |   |   |
| 11. $a^{3m} + 3a^{2m} b^n + 3a^m b^{2n} + b^{3n}$ .               |   |   |
| 12. $m^{2a} + 2m^a n^b + 2m^a p^c + n^{2b} + 2n^b p^c + p^{2c}$ . |   |   |
| 13. $y^{2n+4}$ .  | 29. $\frac{a-8}{x-y}$ .                         | 46. $m^{5n} p^5$ .  |
| 14. $d^{2n+4}$ .  | 30. $-\frac{a-b}{x-y}$ .                        | 47. $25a^{2n-3} b^4$ .  |
| 15. $c^{2(m+n)}$ .  | 31. $\frac{a-b}{x-y}$ .                         | 48. $27a^9 b^{3n-6}$ .  |
| 16. $-a^{2m+7}$ .   | 32. $-\left(\frac{x^4-y^4}{a^2-b^2}\right)^2$ . | 49. $343a^6 x^{3n-6} y^{3m+3}$ .                                |
| 17. $-m^{2p+7}$ .   | 33. $\frac{x^5-y^5}{a^2-b^2}(x^3+y^3)^3$ .      | 50. $512a^{3n} b^{3m-3} c^{3n-6}$ .                             |
| 18. $x^{2m+6}$ .  | 34. $(x^5+y^5)^m \times (m^8-n^8)^m$ .          | 51. $\frac{a^4 b^6}{c^8 d^{10}}$ .                              |
| 19. $x^{3n+2} y^{2m+2}$ .   | 35. $a^{12}$ .                                  | 52. $a^3 x^7$ .   |
| 20. $-\frac{x^{15}}{y^{15}}$ .                                    | 36. $b^{30}$ .                                  |   |
| 21. $\frac{a^{10}}{b^{10}}$ .                                     | 37. $x^{2m+2}$ .                                | 53. $\frac{1}{b^3}$ .   |
| 22. $-\frac{n}{m}$ .  | 38. $a^6$ .                                     | 54. $\frac{1}{a^8 b^2}$ .                                       |
| 23. $-\frac{b^{12}}{x^3 c^9}$ .                                   | 39. $-a^6$ .                                    | 55. $abcde$ .   |
| 24. $-\frac{x}{y}$ .  | 40. $-m^{21}$ .                                 | 56. $\left(\frac{8x-6y}{4x-3y}\right)^{10a^2}$<br>$= 1024a^2$ . |
| 25. $\frac{n^3}{m^2}$ .   | 41. $y^{6m}$ .                                  | 57. $\frac{m^{16} q^{39}}{n^{12} p^2}$ .                        |
| 26. $-\frac{a^{19} x^3}{b^3 y^{19}}$ .                            | 42. $-y^{6m}$ .                                 | 58. $\frac{a^{20} c^3 x^6}{b^9}$ .                              |
| 27. $\left(\frac{m-5}{7-c}\right)^2$ .                            | 43. $y^{6m}$ .                                  |   |
| 28. $\left(\frac{x-y}{m-p}\right)^2$ .                            | 44. $-b^{4m-2}$ .                               |   |
|   | 45. $-b^{2n(n-1)}$ .                            |   |

## Ex. 36. Page 136.

- |   |                         |   |
|---|-------------------------|---|
| 1. $c^{s-1}$ .  | 5. $x^{s-3s}$ .         | 9. $(x+y)^{2(a-b)}$ .                     |
| 2. $m^s$ .  | 6. $x^{3s}$ .           | 10. $a^{p-q}b^{p+q}$ .                    |
| 3. $m^{s-1}$ .  | 7. $x^{2(a-b)}$ .       | 11. $x^{3p}+x^{2p}+x^p+1$ .               |
| 4. $m^s$ .  | 8. $a^{s+5y}$ .         | 12. $2-x^p+3x^{2p}$ .                     |
| 13. $a^{n-1}+a^{n-2}b+a^{n-3}b^2+\dots+ab^{n-2}+b^{n-1}$ .      |                         |   |
| 14. $a^{2n-1}-a^{2n-2}b+a^{2n-3}b^2-\dots+ab^{2n-2}-b^{2n-1}$ . |                         |   |
| 15. $a^n$ .   | 34. $\frac{1}{m^3}$ .   | 52. $-\frac{1}{x^3}$ .                    |
| 16. $(\frac{2}{3}ac)^n$ .                                       | 35. $\frac{1}{m^3}$ .   | 53. $-\frac{1}{x^{16}}$ .                 |
| 17. $(5a^2-3b^2)^x$ .   | 36. $b^m$ .             | 54. $a^5$ .                               |
| 18. $(8m+7n)^b$ .   | 37. $b^m$ .             | 55. $m^3$ .                               |
| 19. 64.   | 38. $\frac{b^m}{d^5}$ . | 56. $-1$ .                                |
| 20. 64.   | 39. 1.                  | 57. 1.                                    |
| 21. 243.  | 40. 1.                  | 58. 1.                                    |
| 22. $\frac{2mn^6}{15pq^6}$ .                                    | 41. 1.                  | 59. $\frac{1}{b^n}$ .                     |
| 23. $\frac{p-q}{27(m+n)^3}$ .                                   | 42. 2.                  | 60. $b^n$ .                               |
| 24. 1.  | 43. 2.                  | 61. $\frac{1}{x^n}$ .                     |
| 25. $\frac{1}{a^3}$ .   | 44. 1.                  | 62. $\frac{1}{x^n}$ .                     |
| 26. 1.  | 45. 0.                  | 63. $\frac{n^3}{m^5}$ .                   |
| 27. $a^4$ .   | 46. 2.                  | 64. $\left(\frac{b}{a}\right)^n$ .        |
| 28. $\frac{1}{x^0}=1$ .   | 47. 1.                  | 65. $\frac{b^n}{a^m}$ .                   |
| 29. 1.  | 48. $\infty$ .          | 66. $\frac{1}{m}$ .                       |
| 30. $a^3$ .   | 49. $\infty$ .          |   |
| 31. $a^3$ .   | 50. $\frac{1}{a^2}$ .   |   |
| 32. $a^5 \times q^0 = a^5$ .                                    | 51. $\frac{1}{x^4}$ .   |   |
| 33. $\frac{1}{m^3}$ .   |                         |   |
|   |                         | 67. $a^{n+1}$ .                           |
|   |                         | 68. $\frac{p}{p^m} = \frac{1}{p^{m-1}}$ . |
|   |                         | 69. $\frac{a^m b^3}{a^2 b^n}$ .           |
|   |                         | 70. $n^4$ .                               |
|   |                         | 71. $\frac{y}{x}$ .                       |
|   |                         | 72. 1.                                    |
|   |                         | 73. 1.                                    |
|   |                         | 74. $\frac{1}{a}$ .                       |
|   |                         | 75. $q^3$ .                               |
|   |                         | 76. $\frac{m}{1-m}$ .                     |
|   |                         | 77. $n$ .                                 |
|   |                         | 78. $\frac{1}{1-x}$ .                     |
|   |                         | 79. $\left(\frac{1}{1-y}\right)^3$ .      |
|   |                         | 80. $(m-n)^4$ .                           |
|   |                         | 81. $\frac{1}{(1-a)^3}$ .                 |
|   |                         | 82. $\frac{2}{3}abc$ .                    |

83.  $\frac{y^4}{5x^2}$  84.  $\frac{1}{b^2c^3}$  85.  $\left(\frac{y}{x}\right)^3$  86.  $a^3$  87.  $\frac{m^2n^3}{mn^2}$
88.  $\frac{a^6}{a^{4a}}$  91.  $\left(\frac{x^m y^n}{x^3 y^n + x^m y^4}\right)^2$
89.  $\left(\frac{m^5 n^3}{m^{2x} n^3 - m^6 n^{6x}}\right)^n$  92.  $\frac{a^4 b^4}{(b+a)^3(b-a)}$
90.  $\left(\frac{b^{2y} - a^{2x}}{a^{2x} b^{2y}}\right)$  93.  $\frac{(n+m)^2(n^2 - m^2)}{m^4 n^4}$
94.  $3m^{-1}$  96.  $-xz^{-3}$  98.  $-bp^{-4}$
95.  $-xy^{-3}$  97.  $bp^{-4}$  99.  $a^5 x a^{-3} = a^2$
100.  $a^{3m}(xy^3)^{-3}$  104.  $(x^{-2} + y^{-1})^{-2} \times (x^{-3} - y^{-4})^{-3}$
101.  $b^m b^{-1} y^{-5} = b^{m-1} y^{-5}$  105.  $(x \pm y)^{-2}(x^2 + y^2)$
102.  $c^4 c^{-m} z^{-p} = c^{4-m} z^{-p}$  106.  $3^{-1}(mn)^2$
103.  $x^{m-5} + y^{n-3}$  107. 1. 108. 2.
109. 18. 116.  $\frac{4}{3}$  123. 400. 130.  $\frac{1}{4}$
110.  $-\frac{5}{84}$  117. 1. 124. 80. 131. 0.
111.  $-375$  118. 1. 125. 405. 132. 3119.
112. 1. 119. 1. 126.  $-20$  133. 525.
113.  $\frac{1}{2}$  120. 1. 127. 20. 134.  $-17$ .
114. 1. 121. 4000. 128. 5. 135. 1.
115.  $\frac{1}{25}$  122. 100,000. 129. 16. 136. 20.

## Ex. 37. Page 138.

1.  $a$ . 7.  $b^{-1}$ . 13.  $x^{-(m+p)}$ . 19.  $a^{-2}$ .
2.  $m^4$ . 8.  $m^2 n^{-2}$ . 14.  $y^{-(a+c)}$ . 20.  $-b^{-3}$ .
3.  $x^4$ . 9.  $a^3 b^{n-m}$ . 15.  $x^{-5m}$ . 21.  $-x^{-7}$ .
4.  $z^{-c}$ . 10.  $a^{-(m+n)}$ . 16.  $y^{-6x}$ . 22.  $y^{-10}$ .
5.  $a^{-1}$ . 11.  $a^{-7}$ . 17.  $a^{-(2b+1)}$ . 23.  $-m^{-5}$ .
6.  $h^{m-n}$ . 12.  $b^{-7}$ . 18.  $a^{-2(x+y)}$ . 24.  $x^{-4}$ .

25.  $x^{-9}$ .      29.  $-y^{-10} - y^{-9} - y^{-8} - y^{-7} - y^{-6} - y^{-5}$ .  
 26.  $-x^{-9}$ .      30.  $\left(\frac{a}{b}\right)^{-3}$ .      31.  $\frac{y^{-3}}{x^{-3}}$ .      32.  $\frac{m^{-1}n^{-2}}{b^{-3}}$ .  
 27.  $y^{-2m}$ .  
 28.  $a^{-7} + a^{-8} + a^{-5} + a^{-4}$ .      33.  $x^{-1} - x^{-2} + 5x^{-4} - 3x^{-5}$ .  
 34.  $12^{-2} = \frac{1}{144}$ .      43. 4.      51.  $\left(\frac{a^4}{b^3} - \frac{m^{-3}}{n^{-4}}\right)^{-1}$ .  
 35.  $30^{-1} = \frac{1}{30}$ .      44. 10,000.      52.  $\left(\frac{x^3}{y^6} - \frac{z^{-3}}{y^{-8}}\right)^{-1}$ .  
 36.  $40^{-2} = \frac{1}{1600}$ .      45.  $(ac)^{-1}$ .      53.  $\left(\frac{9x^2}{a^2} - \frac{4m^{-3}}{b^{-1}}\right)^{-1}$ .  
 37.  $6^{-3} = \frac{1}{216}$ .      46.  $(\frac{1}{3}xy)^{-4}$ .      54.  $\left(\frac{a^{-3}}{z^4} - \frac{x^{-6}}{y^{-8}}\right)^{-3}$ .  
 38.  $10^{-4} = \frac{1}{10000}$ .      47.  $(a-b)^{-8}$ .      55.  $\left(\frac{4}{y^{-6}} - \frac{1}{z^2}\right)^{-6}$ .  
 39.  $\frac{1}{9}$ .      48.  $(x^2 - y^2)^{-16}$ .  
 40.  $\frac{1}{9}$ .      49.  $(3m^5 - 2n)^{-4}$ .  
 41.  $\frac{1}{27}$ .      50.  $-(1 - x^2)^{-14}$ .  
 42.  $\frac{1}{27}$ .

## Ex. 38. Page 140.

1.  $2^{-6} = \frac{1}{64}$ .      14.  $\frac{a^{-4}}{256}$ .      25.  $-x^{10}y^{20}$ .  
 2.  $2^{-6} = \frac{1}{64}$ .      15.  $x^8$ .      26.  $x^2y^4$ .  
 3.  $5^{12} = 244,140,625$ .      16.  $y^{-4}$ .      27.  $a^{mn}$ .  
 4.  $\frac{1}{10000000000}$ .      17. 1.      28.  $b^{-3m}$ .  
 5. 729.      18.  $a^{-2}$ .      29.  $d^{5p}$ .  
 6. 125.      19.  $b^{-4}$ .      30.  $c^{-2x}$ .  
 7. 4096.      20.  $-m^{-3}$ .      31.  $m^{-2xy}$ .  
 8. 1,000,000.      21.  $a^{-4}b^3$ .      32.  $a^{-2y}$ .  
 9.  $3\frac{3}{8}$ .      22.  $mn^3$ .      33.  $a^{2m}b^{-2x}$ .  
 10. 625.      23.  $a^{-4}b^{-10}$ .      34.  $b^3c^{2y}$ .  
 11.  $\frac{2}{9}$ .      24.  $-a^3b^6$ .      35.  $x^{-2am}y^{-2mx}$ .  
 12.  $3\frac{3}{4}$ .      36.  $(3a)^{4x}$ .  
 13.  $64a^6$ .      37.  $-a^{-6}$ .

- |   |  |                              |
|---|--|------------------------------|
| 38. $a^{-4n(2n+1)}$ .                             | 53. $\left(\frac{c-d}{a+b}\right)^2$ . | 72. 0.49.                    |
| 39. $a^{-6n}$ .                                   | 54. $\left(\frac{y-x}{a+b}\right)$ .   | 73. $15\frac{1}{2}$ .        |
| 40. $m^{-2pqs}$ .                                 | 55. $\frac{c-d}{m-n}$ .                | 74. 1000.                    |
| 41. $\frac{a^2b^{-4}}{c^{2m}d^{2n}}$ .            | 56. 2.                                 | 75. 125.                     |
| 42. $\frac{b^{-m}c^{3m}}{x^{2m}y^{4m}}$ .         | 57. 27.                                | 76. $a^2$ .                  |
| 43. $\frac{xy^{-1}}{a^{-1}}$ .                    | 58. 25.                                | 77. $\frac{1}{a}$ .          |
| 44. $\frac{m^{-1}}{x^py^q}$ .                     | 59. $\frac{1}{49}$ .                   | 78. $\frac{1}{a^2}$ .        |
| 45. $\frac{m-n}{m+n}$ .                           | 60. $\frac{1}{1296}$ .                 | 79. $m^2$ .                  |
| 46. $\left(\frac{m+n}{p+q}\right)^{-2}$ .         | 61. $\frac{1}{106}$ .                  | 80. $\frac{m^2}{m^2}$ .      |
| 47. $\left(\frac{a+b}{a-b}\right)^2$ .            | 62. $\frac{1}{2401}$ .                 | 81. $x^2$ .                  |
| 48. $\left(\frac{x-y}{x+y}\right)$ .              | 63. 81.                                | 82. $\frac{y^m}{y^m}$ .      |
| 49. $\frac{m+n}{m-n}$ .                           | 64. $\frac{1}{81}$ .                   | 83. $b^2$ .                  |
| 50. $1-x^2$ .                                     | 65. $\frac{1}{16807}$ .                | 84. $\frac{1}{c^{6a}}$ .     |
| 51. 1.  | 66. 1.                                 | 85. $\frac{1}{(a+b)^{2y}}$ . |
| 52. $\left(\frac{x+y}{a-b}\right)$ .              | 67. $\frac{1}{9}$ .                    | 86. $\frac{ab^2n}{m^4}$ .    |
|   | 68. $\frac{8}{27}$ .                   | 87. $a^{-3}+b^{-2}$ .        |
|   | 69. $\frac{1}{82}$ .                   |                              |
|   | 70. $15\frac{1}{2}$ .                  |                              |
|   | 71. 0.09.                              |                              |
| 88. $-\frac{1}{x^m}(x^{m-1}+x^{m-2}+\dots+x+1)$ . |  |                              |
| 89. $-x^m-x^{m-1}-x^{m-2}-\dots$ .                |  |                              |
| 90. 256.  | 92. $\frac{1}{8}$ .                    | 94. $\frac{1}{81}$ .         |
| 91. $\frac{82}{243}$ .                            | 93. 32.                                | 95. 128.                     |



- |                       |                           |                           |                                      |
|-----------------------|---------------------------|---------------------------|--------------------------------------|
| 96. $\frac{1}{81}$ .  | 99. $\frac{1}{1000000}$ . | 102. $\frac{9}{16}$ .     | 105. 1.                              |
| 97. $15\frac{5}{8}$ . | 100. $\frac{8}{27}$ .     | 103. 1.                   | 106. $\frac{1}{a^{10}b^{14}}$ .      |
| 98. $\frac{4}{81}$ .  | 101. $\frac{8}{11}$ .     | 104. $\frac{248}{1024}$ . | 107. $\frac{1}{x^3} - \frac{1}{y}$ . |

## Ex. 39. Page 143.

- |  |  |                                       |                                       |               |
|--|--|---------------------------------------|---------------------------------------|---------------|
| 1. $a^{\frac{1}{2}}$ .                           | 3. $a^3$ .   | 5. $a^{\frac{1}{2}}$ .                | 7. $ab^{\frac{1}{2}}$ .               | 9. $a - b$ .  |
| 2. $a^{\frac{1}{2}}$ .                           | 4. $n^{\frac{1}{2}}$ .   | 6. $a^{\frac{1}{2}}b^{\frac{1}{2}}$ . | 8. $a^{\frac{1}{2}}b^{\frac{1}{2}}$ . | 10. $m - p$ . |
| 11. $\frac{3b^{\frac{1}{2}}}{a^{\frac{1}{2}}}$ . | 29. $\sqrt[4]{\frac{1}{c^3}}$ .                                | 41. 2.                                | 60. $\frac{1}{16}$ .                  |               |
| 12. $\frac{m^{\frac{1}{2}}}{n^{\frac{1}{2}}}$ .  | 30. $\sqrt[n]{\frac{1}{e^m}}$ .                                | 42. 3.                                | 61. $\frac{1}{64}$ .                  |               |
| 13. $\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}$ .  | 31. $\sqrt[3]{\frac{1}{x^{2m}}}$ .                             | 43. 5.                                | 62. $\frac{1}{81}$ .                  |               |
| 14. $x^{\frac{1}{2}}$ .                          | 32. $\sqrt[m+1]{\frac{1}{y^{a+b}}}$ .                          | 44. 2.                                | 63. $\frac{1}{8}$ .                   |               |
| 15. $m^{\frac{1}{2}}$ .                          | 33. $\frac{z^p}{\sqrt[3]{z}} = \sqrt[3]{z^{3p-1}}$ .           | 45. 3.                                | 64. $\frac{1}{8}$ .                   |               |
| 16. $a^{\frac{1}{2}}$ .                          | 34. $\sqrt[3]{(x^3 + y^3)^2}$ .                                | 46. 2.                                | 65. $\frac{1}{125}$ .                 |               |
| 17. $a^{\frac{1}{2}}$ .                          | 35. $\sqrt{x^5y^3}$ .  | 47. 5.                                | 66. $\frac{5}{8}$ .                   |               |
| 18. $\sqrt[3]{a}$ .                              | 36. $\frac{1}{\sqrt[3]{m^3 - n^3}}$ .                          | 48. 6.                                | 67. $\frac{2}{3}$ .                   |               |
| 19. $\sqrt[5]{1} = 1$ .                          | 37. $\frac{\sqrt{a}}{b}$ .                                     | 49. 7.                                | 68. $\frac{5}{8}$ .                   |               |
| 20. $\sqrt[4]{m^3}$ .                            | 38. $\frac{m^2}{\sqrt[5]{n^8}}$ .                              | 50. 9.                                | 69. $\frac{2}{3}$ .                   |               |
| 21. $\sqrt[7]{x^3}$ .                            | 39. $\sqrt[7]{\frac{x^{15}}{y^{20}}}$ .                        | 51. 32.                               | 70. $\frac{1}{2}$ .                   |               |
| 22. $\sqrt[3]{n^2}$ .                            | 40. $\sqrt{\frac{1}{a^6b^3}} = \frac{1}{a^3b^{\frac{3}{2}}}$ . | 52. 729.                              | 71. $\frac{6}{5}$ .                   |               |
| 23. $\sqrt[4]{b^3}$ .                            |  | 53. 3125.                             | 72. $\frac{3}{2}$ .                   |               |
| 24. $\sqrt[6]{e^5}$ .                            |  | 54. 16.                               | 73. $\frac{6}{5}$ .                   |               |
| 25. $\sqrt[9]{h}$ .                              |  | 55. 243.                              | 74. $\frac{3}{2}$ .                   |               |
| 26. $\sqrt[6]{x^a}$ .                            |  | 56. 64.                               | 75. 2.                                |               |
| 27. $\sqrt[m]{y}$ .                              |  | 57. 64.                               | 76. $\frac{4}{9}$ .                   |               |
| 28. $\sqrt[3]{\frac{1}{a}}$ .                    |  | 58. 16.                               | 77. $\frac{2}{5}$ .                   |               |
|  |  | 59. $\frac{1}{8}$ .                   | 78. $\frac{27}{8}$ .                  |               |

79.  $\frac{8}{27}$ .      81. 0.09.      82.  $\frac{1}{0.5} = 2$ .      83. +0.09.  
80. 0.5.      84. -0.008.

## Ex. 40. Page 145.

1.  $4a^{\frac{1}{2}}$ .      5.  $0.81m^{-\frac{1}{2}} = \left(\frac{0.9}{m}\right)^2$ .  
2.  $\frac{b^{\frac{1}{2}}}{2} + 4b^{-\frac{1}{2}} = \frac{b^{\frac{1}{2}} + 8}{2b^{\frac{1}{2}}}$ .      6.  $0.4c^{-\frac{1}{2}}$ .  
3.  $5(m^{\frac{1}{2}} - 2m^{-\frac{1}{2}})$ .      7.  $-1.3d^{-4}$ .  
4.  $9x^{-3}$ .      8.  $x^{-4}y^{\frac{1}{2}} - 2x^{-3}y^{\frac{1}{2}}$ .  
9.  $-(m+1)x^{-\frac{1}{2}} + (a+d)x^{-\frac{1}{2}} + (3b-c)x^{-\frac{1}{2}}$ .  
10.  $(2-m)y^{-\frac{1}{2}} + (5b+m-c)y^{-\frac{1}{2}} + (3-c)y^{\frac{1}{2}}$ .  
11.  $-(a+1)b^{-\frac{1}{2}}$ .      13.  $0.5e^{-x}$ .      15.  $4m^{-5}$ .  
12.  $-3.7d^{-\frac{1}{2}}$ .      14.  $-1.15h^{-4}$ .      16.  $1.3p^{-3}$ .  
17.  $2.4x^{-1} - 7.7x^{-2} + 0.1x^{-3}$ .  
18.  $a^{\frac{1}{2}}$ .      24.  $m^{-\frac{1}{2}}$ .      29.  $z^{-\frac{c}{m}}$ .      33.  $a^{\frac{1}{2}}$ .  
19.  $b^{\frac{1}{2}}$ .      25.  $x^{\frac{(m-n)}{mn}}$ .      34.  $b^{\frac{1}{2}}$ .  
20.  $a^{-\frac{1}{2}}$ .      26.  $x^{\frac{mq+np}{nq}}$ .      35.  $h^{\frac{1}{2}}$ .  
21.  $c^{-\frac{1}{2}}$ .      27.  $y^{\frac{(am+cn)}{cn}}$ .      36.  $m^{\frac{a}{bx}}$ .  
22.  $d^{-\frac{1}{2}}$ .      28.  $a^{-\frac{x}{y}}$ .      32.  $e^{\frac{-x(2y+1)}{y}}$ .      37.  $n^{-\frac{ac}{bd}}$ .  
23.  $m^{-1}$ .      38.  $x^r + 2x^{\frac{r}{2}}y^{2p} + 9y^{4p}$ .      39.  $x^p + x^{\frac{p}{2}}y^{\frac{q}{2}} + y^q$ .  
40.  $x^{-2} + x^{-1}y^{-1} + y^{-2}$ .  
41.  $c^{-\frac{7}{m}} - 5c^{-\frac{6}{m}} + 10c^{-\frac{5}{m}} - 8c^{-\frac{4}{m}} - 3c^{-\frac{3}{m}} + c^{-\frac{2}{m}}$ .  
42.  $d^{-\frac{2}{x}} - \frac{5}{6}d^{-\frac{3}{x}} + \frac{1}{2}d^{-\frac{4}{x}} + 2\frac{2}{3}d^{-\frac{5}{x}} - 1\frac{2}{3}d^{-\frac{6}{x}} + \frac{4}{3}d^{-\frac{7}{x}}$ .  
43.  $\frac{25x^{-\frac{1}{2}}}{16y^{-\frac{1}{2}}} - \frac{9a^{-1}}{4b^{-\frac{1}{2}}}$ .      44.  $4a^{-2}b^{-\frac{1}{2}} - 12a^{-\frac{1}{2}}b^{-\frac{1}{2}} + 9a^{-\frac{1}{2}}b^{-\frac{3}{2}}$ .  
45.  $e^{-\frac{1}{2}}$ .      46.  $h^{-\frac{1}{2}}$ .      47.  $k^{-\frac{1}{2}}$ .

- |                               |                                |   |
|-------------------------------|--------------------------------|---|
| 48. $l^{\frac{1}{2}}$ .       | 55. $t^{-\frac{(am-n)}{an}}$ . | 61. $d^{\frac{b-2a}{2m}}$ .   |
| 49. $g^{\frac{1}{2}}$ .       | 56. $v^{-\frac{(b+ap)}{bp}}$ . | 62. $e^{\frac{am+n}{an}}$ .   |
| 50. $a^{\frac{1}{2}}$ .       | 57. $x^{\frac{am-bm}{bm}}$ .   | 63. $a^{\frac{1}{2}}b^{-\frac{3p}{2}}$ .                                      |
| 51. $b^{-\frac{1}{2}}$ .      | 58. $z^{-\frac{(aq+p)}{q}}$ .  | 64. $\frac{a^{-\frac{1}{2}}}{3b^{-\frac{1}{2}}}$ .                            |
| 52. $m^{-\frac{1}{2}}$ .      | 59. $y^{\frac{m-n}{n}}$ .      | 65. $x+y$ .   |
| 53. $p^{-\frac{(m+n)}{mn}}$ . | 60. $c^{\frac{a-2}{m}}$ .      | 66. $x-x^{\frac{1}{2}}$ .   |
| 54. $q^{\frac{a+b}{ab}}$ .    |                                | 67. $x^{\frac{1}{2}}-\frac{2}{3}x^{\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}$ . |

## Ex. 41. Page 147.

- $a^3 + 3a^2b + 3ab^2 + b^3$ .
- $m^5 + 5m^4n + 10m^3n^2 + 10m^2n^3 + 5mn^4 + n^5$ .
- $c^8 + 8c^7d + 28c^6d^2 + 56c^5d^3 + 70c^4d^4 + 56c^3d^5 + 28c^2d^6 + 8cd^7 + d^8$ .
- $h^{10} + 10h^9l + 45h^8l^2 + 120h^7l^3 + 210h^6l^4 + 252h^5l^5 + 210h^4l^6 + 120h^3l^7 + 45h^2l^8 + 10hl^9 + l^{10}$ .
- $x^{11} + 11x^{10}y + 55x^9y^2 + 165x^8y^3 + 330x^7y^4 + 462x^6y^5 + 462x^5y^6 + 330x^4y^7 + 165x^3y^8 + 55x^2y^9 + 11xy^{10} + y^{11}$ .
- $y^7 - 7y^6z + 21y^5z^2 - 35y^4z^3 + 35y^3z^4 - 21y^2z^5 + 7yz^6 - z^7$ .
- $x^9 - 9x^8a + 36x^7a^2 - 84x^6a^3 + 126x^5a^4 - 126x^4a^5 + 84x^3a^6 - 36x^2a^7 + 9xa^8 - a^9$ .
- $y^3 - 3y^2b + 3yb^2 - b^3$ .
- $c^5 - 5c^4d + 10c^3d^2 - 10c^2d^3 + 5cd^4 - d^5$ .
- $m^8 - 8m^7a + 28m^6a^2 - 56m^5a^3 + 70m^4a^4 - 56m^3a^5 + 28m^2a^6 - 8ma^7 + a^8$ .
- $1 + 4a + 6a^2 + 4a^3 + a^4$ .

12.  $a^4 - 4a^3 + 6a^2 - 4a + 1.$
13.  $1 + 6d + 15d^2 + 20d^3 + 15d^4 + 6d^5 + d^6.$
14.  $1 - 8h + 28h^2 - 56h^3 + 70h^4 - 56h^5 + 28h^6 - 8h^7 + h^8.$
15.  $1 + 10x + 45x^2 + 120x^3 + 210x^4 + 252x^5 + 210x^6$   
 $+ 120x^7 + 45x^8 + 10x^9 + x^{10}.$
16.  $32 + 80a + 80a^2 + 40a^3 + 10a^4 + a^5.$
17.  $729 - 1458a + 1215a^2 - 540a^3 + 135a^4 - 18a^5 + a^6.$
18.  $m^9 - 18m^8 + 144m^7 - 672m^6 + 2106m^5 - 4032m^4$   
 $+ 5376m^3 - 4608m^2 + 2304m - 512.$
19.  $x^3 - 15x^2 + 75x - 125.$
20.  $y^5 - 15y^4 + 90y^3 - 270y^2 + 405y - 243.$
21.  $a^{21} + 7a^{18}b^3 + 21a^{15}b^4 + 35a^{12}b^6 + 35a^9b^8 + 21a^6b^{10}$   
 $+ 7a^3b^{12} + b^{14}.$
22.  $m^{40} - 8m^{35}n^5 + 28m^{30}n^4 - 56m^{25}n^6 + 70m^{20}n^8 - 56m^{15}n^{10}$   
 $+ 28m^{10}n^{12} - 8m^5n^{14} + n^{16}.$
23.  $x^{15} + 5x^{12}y^4 + 10x^9y^8 + 10x^6y^{12} + 5x^3y^{16} + y^{20}.$
24.  $a^{32} - 8a^{28}y^4 + 28a^{24}y^6 - 56a^{20}y^8 + 70a^{16}y^{12} - 56a^{12}y^{15}$   
 $+ 28a^8y^{18} - 8a^4y^{21} + y^{24}.$
25.  $a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^{10}$   
 $+ 7ab^{12} - b^{14}.$
26.  $\frac{a^2}{b^3} + \frac{2ax}{by} + \frac{x^2}{y^3}.$
27.  $\frac{a^3}{c^3} - \frac{3a^2d}{cx^2} + \frac{3ad^2}{cx^3} - \frac{d^3}{x^3}.$
28.  $\frac{m^5}{n^5} + \frac{5m^4p}{n^4q} + \frac{10m^3p^2}{n^3q^2} + \frac{10m^2p^3}{n^2q^3} + \frac{5mp^4}{nq^4} + \frac{p^5}{q^5}.$
29.  $\frac{m^6}{p^6} - \frac{6m^5n}{p^5q} + \frac{15m^4n^2}{p^4q^2} - \frac{20m^3n^3}{p^3q^3} + \frac{15m^2n^4}{p^2q^4} - \frac{6mn^5}{pq^5} + \frac{n^6}{q^6}.$
30.  $\frac{1}{a^4} - \frac{4b}{a^3c} + \frac{6b^2}{a^2c^2} - \frac{4b^3}{ac^3} + \frac{b^4}{c^4}.$

31.  $m^{\frac{1}{2}} - 5m^{\frac{1}{2}}n^{\frac{1}{2}} + 10m^{\frac{1}{2}}n - 10m^{\frac{1}{2}}n^{\frac{1}{2}} + 5m^{\frac{1}{2}}n^2 - n^{\frac{1}{2}}$ .
32.  $a^{\frac{1}{2}} + 6a^{\frac{1}{2}}b^{\frac{1}{2}} + 15a^{\frac{1}{2}}b^{\frac{1}{2}} + 20a^{\frac{1}{2}}b^2 + 15a^{\frac{1}{2}}b^{\frac{3}{2}} + 6a^{\frac{1}{2}}b^{\frac{5}{2}} + b^{\frac{1}{2}}$ .
33.  $x^{\frac{1}{2}} + 7x^{\frac{1}{2}}y^{\frac{1}{2}} + 21x^{\frac{1}{2}}y^{\frac{1}{2}} + 35x^{\frac{1}{2}}y^{\frac{1}{2}} + 35x^{\frac{1}{2}}y + 21x^{\frac{1}{2}}y^{\frac{1}{2}} + 7x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{\frac{1}{2}}$ .
34.  $x^2 - 4xz^{\frac{1}{2}} + 6xz^{\frac{1}{2}} - 4xz + z^{\frac{1}{2}}$ .
35.  $a + 5a^{\frac{1}{2}}b^{\frac{1}{2}} + 10a^{\frac{1}{2}}b^{\frac{1}{2}} + 10a^{\frac{1}{2}}b^{\frac{1}{2}} + 5a^{\frac{1}{2}}b^{\frac{1}{2}} + b^{\frac{1}{2}}$ .
36.  $a^{-5} + 5a^{-4}b^{-2} + 10a^{-3}b^{-4} + 10a^{-2}b^{-6} + 5a^{-1}b^{-8} + b^{-10}$ .
37.  $a^{-24} + 8a^{-21}x^{-1} + 28a^{-18}x^{-2} + 56a^{-15}x^{-3} + 70a^{-12}x^{-4} + 56a^{-9}x^{-5} + 28a^{-6}x^{-6} + 8a^{-3}x^{-7} + x^{-8}$ .
38.  $m^{-28} - 7m^{-24}n^{-1} + 21m^{-20}n^{-2} - 35m^{-16}n^{-3} + 35m^{-12}n^{-4} - 21m^{-8}n^{-5} + 7m^{-4}n^{-6} - n^{-7}$ .
39.  $d^{-12} - 6d^{-10}x^2 + 15d^{-8}x^4 - 20d^{-6}x^6 + 15d^{-4}x^8 - 6d^{-2}x^{10} + x^{12}$ .
40.  $c^{-20} + 4c^{-15}z + 6c^{-10}z^2 + 4c^{-5}z^3 + z^4$ .
41.  $m^{-\frac{1}{2}} + 5m^{-2}n^{-\frac{1}{2}} + 10m^{-\frac{1}{2}}n^{-\frac{1}{2}} + 10m^{-1}n^{-2} + 5m^{-\frac{1}{2}}n^{-\frac{1}{2}} + n^{-\frac{1}{2}}$ .
42.  $n^{-\frac{1}{2}} + 5n^{-3}p^{-\frac{1}{2}} + 10n^{-\frac{1}{2}}p^{-\frac{1}{2}} + 10n^{-\frac{1}{2}}p^{-2} + 5n^{-\frac{1}{2}}p^{-\frac{1}{2}} + p^{-\frac{1}{2}}$ .
43.  $n^{-\frac{1}{2}} - 4n^{-\frac{1}{2}}p^{-\frac{1}{2}} + 6n^{-\frac{1}{2}}p^{-\frac{1}{2}} - 4n^{-\frac{1}{2}}p^{-\frac{1}{2}} + p^{-\frac{1}{2}}$ .
44.  $a^{-\frac{1}{2}} + 5a^{-2}b^{-\frac{1}{2}} + 10a^{-\frac{1}{2}}b^{-\frac{1}{2}} + 10a^{-1}b^{-1} + 5a^{-\frac{1}{2}}b^{-\frac{1}{2}} + b^{-\frac{1}{2}}$ .
45.  $x^{-\frac{1}{2}} + 3x^{-\frac{1}{2}}y + 3x^{-\frac{1}{2}}y^2 + y^{\frac{1}{2}}$ .
46.  $y^{-12} - 6y^{-10}z^{-\frac{1}{2}} + 15y^{-8}z^{-\frac{1}{2}} - 20y^{-6}z^{-2} + 15y^{-4}z^{-\frac{1}{2}} - 6y^{-2}z^{-\frac{1}{2}} + z^{-4}$ .
47.  $u^{-\frac{1}{2}} + 4u^{-\frac{1}{2}}v^3 + 6u^{-\frac{1}{2}}v^6 + 4u^{-\frac{1}{2}}v^9 + v^{12}$ .
48.  $1 - 5z^{-\frac{1}{2}} + 10z^{-\frac{1}{2}} - 10z^{-2} + 5z^{-\frac{1}{2}} - z^{-\frac{1}{2}}$ .
49.  $16x^4 + 32x^2y + 24x^2y^2 + 8xy^3 + y^4$ .
50.  $243y^5 - 405y^4z + 270y^3z^2 - 90y^2z^3 + 15yz^4 - z^5$ .

51.  $4096z^6 - 6144z^5u + 3840z^4u^2 - 1280z^3u^3 + 240z^2u^4 - 24zu^5 + u^6.$
52.  $128a^7 - 448a^6b^2 + 672a^5b^4 - 560a^4b^6 + 280a^3b^8 - 84a^2b^{10} + 14ab^{12} - b^{14}.$
53.  $m^{12} + 18m^{10}b + 135m^8b^2 + 540m^6b^3 + 1215m^4b^4 + 1458m^2b^5 + 729b^6.$
54.  $a^5 - 10a^4c + 40a^3c^2 - 80a^2c^3 + 80ac^4 - 32c^5.$
55.  $x^{12} - 12x^9d + 54x^6d^2 - 108x^3d^3 + 81d^4.$
56.  $u^{-2} - 12u^{-\frac{1}{2}}e + 48u^{-\frac{3}{2}}e^2 - 64e^3.$
57.  $128a^{14} - 1344a^{12}b + 6048a^{10}b^2 - 15,120a^8b^3 + 22,680a^6b^4 - 20,412a^4b^5 + 10,206a^2b^6 - 2187b^7.$
58.  $729x^{-8} + 2916x^{-5}y^3 + 4860x^{-2}y^4 + 4320x^{-3}y^5 + 2160x^{-2}y^6 + 576x^{-1}y^7 + 64y^8.$
59.  $256x^3 + 3072x^2z^{-\frac{1}{2}} + 16,128x^2z^{-1} + 48,384x^2z^{-\frac{3}{2}} + 90,720x^2z^{-2} + 108,864x^2z^{-\frac{5}{2}} + 81,648x^2z^{-3} + 34,992xz^{-\frac{7}{2}} + 6561z^{-4}.$
60.  $\frac{1}{8}(a^5 - 15a^4d^{-1} + 90a^3d^{-2} - 270a^2d^{-3} + 405ad^{-4} - 243d^{-5}).$
61.  $3125m^{-10} + 625m^{-8}n^{-1} + 50m^{-6}n^{-2} + 2m^{-4}n^{-3} + \frac{m^{-2}n^{-4}}{25} + \frac{n^{-5}}{3125}.$
62.  $\frac{64a^6}{b^6} - \frac{576a^5c}{b^5d} + \frac{2160a^4c^2}{b^4d^2} - \frac{4320a^3c^3}{b^3d^3} + \frac{4860a^2c^4}{b^2d^4} - \frac{2916ac^5}{bd^5} + \frac{729c^6}{d^6}.$
63.  $\frac{1}{32a^5} + \frac{15}{16a^4b} + \frac{45}{4a^3b^2} + \frac{135}{2a^2b^3} + \frac{405}{2ab^4} + \frac{243}{b^5}.$
64.  $\frac{16}{a^{-4}} + \frac{32d^{-2}}{a^{-3}c^{-3}} + \frac{24d^{-4}}{a^{-2}c^{-6}} + \frac{8d^{-6}}{a^{-1}c^{-9}} + \frac{d^{-8}}{c^{-12}}.$

$$65. \frac{x^7}{128} - \frac{7x^6y^{\frac{1}{2}}}{192} + \frac{7x^5y^{\frac{3}{2}}}{96} - \frac{35x^4y}{432} + \frac{35x^3y^{\frac{5}{2}}}{648} - \frac{7x^2y^{\frac{7}{2}}}{324} \\ + \frac{7xy^{\frac{9}{2}}}{1458} - \frac{y^{\frac{11}{2}}}{2187}.$$

$$66. \frac{a^5}{243b^5} + \frac{10a^4m^{\frac{1}{2}}}{81b^4x} + \frac{40a^3m^{\frac{3}{2}}}{27b^3x^2} + \frac{80a^2m}{9b^2x^3} + \frac{80am^{\frac{5}{2}}}{3bx^4} + \frac{32m^{\frac{3}{2}}}{x^5}.$$

$$67. 1 - \frac{9y}{2} + \frac{135y^2}{16} - \frac{135y^3}{16} + \frac{1215y^4}{256} - \frac{729y^5}{512} + \frac{729y^6}{4096}.$$

$$68. \frac{2187a^7}{b^7} - \frac{10206a^8}{b^6x} + \frac{20412a^9}{b^5x^2} - \frac{22680a^{10}}{b^4x^3} + \frac{15120a^{11}}{b^3x^4} \\ - \frac{6048a^{12}}{b^2x^5} + \frac{1344a^{13}}{bx^6} - \frac{128a^{14}}{x^7}.$$

$$69. 625m^{4x} - \frac{500m^{3x}n^{2y}}{3} + \frac{50m^{2x}n^{4y}}{3} - \frac{20m^xn^{6y}}{27} + \frac{n^{8y}}{81}.$$

$$70. 243b^{5(m+1)} + 810b^{4(m+1)}a^{2n-1} + 1080b^{3(m+1)}a^{2(n-1)} \\ + 720b^{2(m+1)}a^{2(n-1)} + 240b^{m+1}a^{4(n-1)} + 32a^{5(n-1)}.$$

$$71. 256x^{4(a-b)} - 768x^{3(a-b)}y^{2-b} + 864x^{2(a-b)}y^{2(2-b)} \\ - 432x^{a-b}y^{3(2-b)} + 81y^{4(2-b)}.$$

$$72. 125m^{3(2x-1)} + \frac{225}{4}m^{2(2x-1)}n^{y-2} + \frac{135}{8}m^{2x-1}n^{2(y-2)} + \frac{27}{8}n^{3(y-2)}.$$

$$73. 4a^6b^2 - \frac{20}{3}a^3b^4m + \frac{25}{9}m^2b^6.$$

$$74. \frac{a^{4m}b^{4x}}{256} - \frac{a^{4m+1}b^{4x-1}}{48} + \frac{a^{4m+2}b^{4x-2}}{24} - \frac{a^{4m+3}b^{4x-3}}{27} + \frac{a^{4m+4}b^{4x-4}}{81}.$$

$$75. 32a^{10}b^{5m} + 40a^{m+8}b^{4m+2} + 20a^{2m+6}b^{3m+4} + 5a^{3m+4}b^{2m+6} \\ + \frac{5}{8}a^{4m+2}b^{m+8} + \frac{1}{82}a^{5m}b^{10}.$$

Ex. 42. Page 148.

1.  $\sqrt{8}.$

4.  $\sqrt{84}.$

7.  $\sqrt[3]{2560}.$

2.  $\sqrt{175}.$

5.  $\sqrt[3]{54}.$

8.  $\sqrt[4]{1024}.$

3.  $\sqrt{180}.$

6.  $\sqrt[3]{1029}.$

9.  $\sqrt[5]{96}.$

- |                               |                                   |                                      |
|-------------------------------|-----------------------------------|--------------------------------------|
| 10. $\sqrt[3]{4374}$ .        | 33. $\sqrt[3]{\frac{8m^2}{27}}$ . | 58. $y^2z\sqrt{z}$ .                 |
| 11. $\sqrt[3]{4000}$ .        | 34. $2\sqrt{2}$ .                 | 59. $m^3n^2\sqrt[3]{n}$ .            |
| 12. $\sqrt[3]{7290}$ .        | 35. $2\sqrt{3}$ .                 | 60. $xy\sqrt[4]{xy^3}$ .             |
| 13. $\sqrt{2624}$ .           | 36. $2\sqrt{7}$ .                 | 61. $uv^3\sqrt[3]{uv}$ .             |
| 14. $\sqrt{5a^2}$ .           | 37. $5\sqrt{2}$ .                 | 62. $yz^2\sqrt[4]{y^3}$ .            |
| 15. $\sqrt[4]{2x^2}$ .        | 38. $6\sqrt{2}$ .                 | 63. $ab^{10}\sqrt[10]{a^3b^5}$ .     |
| 16. $\sqrt[5]{6y^5}$ .        | 39. $2\sqrt[3]{9}$ .              | 64. $2a\sqrt{b}$ .                   |
| 17. $\sqrt[3]{7z^3}$ .        | 40. $5\sqrt[3]{4}$ .              | 65. $5m^2\sqrt{x}$ .                 |
| 18. $\sqrt[4]{6b^4}$ .        | 41. $3\sqrt[3]{4}$ .              | 66. $6ab\sqrt{a}$ .                  |
| 19. $\sqrt[5]{-7d^5}$ .       | 42. $4\sqrt[3]{3}$ .              | 67. $4x^2\sqrt[3]{y}$ .              |
| 20. $-\sqrt[6]{10m^6}$ .      | 43. $2\sqrt[3]{7}$ .              | 68. $3m\sqrt[4]{m^3n^3}$ .           |
| 21. $\sqrt[3]{-15n^3}$ .      | 44. $2\sqrt[5]{5}$ .              | 69. $2ab\sqrt[6]{a^3}$ .             |
| 22. $-\sqrt{9a}$ .            | 45. $4\sqrt[5]{2}$ .              | 70. $8c^2\sqrt{5d}$ .                |
| 23. $\sqrt[3]{-125b}$ .       | 46. $3\sqrt[5]{5}$ .              | 71. $\frac{2a}{3}\sqrt{ab}$ .        |
| 24. $\sqrt[5]{-32x}$ .        | 47. $3\sqrt[4]{7}$ .              | 72. $\frac{5}{6n^2}\sqrt[3]{m^2n}$ . |
| 25. $-\sqrt[4]{256y}$ .       | 48. $2\sqrt[4]{7}$ .              | 73. $\frac{a^2}{2}\sqrt{a}$ .        |
| 26. $\sqrt{9bx}$ .            | 49. $18\sqrt{5}$ .                | 74. $\frac{m}{3}\sqrt[5]{mn}$ .      |
| 27. $\sqrt{a^2b}$ .           | 50. $18\sqrt[3]{4}$ .             | 75. $\frac{xy}{6}\sqrt[4]{xy^3}$ .   |
| 28. $\sqrt{m^4yz^3}$ .        | 51. $10\sqrt[4]{9}$ .             | 76. $\frac{m-n}{7}\sqrt{y}$ .        |
| 29. $\sqrt[5]{n^{15}xy^2}$ .  | 52. $a\sqrt{b}$ .                 | 77. $\frac{x-1}{8}\sqrt[3]{z^2}$ .   |
| 30. $-\sqrt[4]{a^{12}m^2n}$ . | 53. $m\sqrt[3]{n^3}$ .            | 78. $\frac{x^3-y^3}{2}\sqrt{x-y}$ .  |
| 31. $\sqrt[3]{-x^{15}y^4z}$ . | 54. $b^2\sqrt[4]{d^3}$ .          |                                      |
| 32. $\sqrt[4]{\frac{a}{4}}$ . | 55. $b\sqrt[4]{c}$ .              |                                      |
|                               | 56. $c^3\sqrt[5]{x^2}$ .          |                                      |
|                               | 57. $x^2y\sqrt[3]{y}$ .           |                                      |



## Ex. 43. Page 150.

1.  $\sqrt[6]{3^3} = \sqrt[6]{27}$ ;  $\sqrt[6]{4}$ .
2.  $\sqrt[4]{6}$ ;  $\sqrt[4]{49}$ .
3.  $\sqrt[6]{125}$ ;  $\sqrt[6]{16}$ .
4.  $\sqrt[12]{2^4} = \sqrt[12]{16}$ ;  $\sqrt[12]{3^3} = \sqrt[12]{27}$ .
5.  $\sqrt[10]{64}$ ;  $\sqrt[10]{32}$ .
6.  $\sqrt[12]{256}$ ;  $\sqrt[12]{512}$ .
7.  $\sqrt[12]{64}$ ;  $\sqrt[12]{625}$ ;  $\sqrt[12]{216}$ .
8.  $\sqrt[20]{16,807}$ ;  $\sqrt[20]{625}$ ;  $\sqrt[20]{14,400}$ .
9.  $\sqrt{3}$ ;  $2\sqrt{3}$ .
10.  $\sqrt{5}$ ;  $2\sqrt{5}$ .
11.  $3\sqrt{7}$ ;  $\sqrt{7}$ .
12.  $6\sqrt{2}$ ;  $2\sqrt{2}$ .
13.  $3\sqrt{11}$ ;  $4\sqrt{11}$ .
14.  $3\sqrt[3]{2}$ ;  $2\sqrt[3]{2}$ .
15.  $2\sqrt[3]{9}$ ;  $3\sqrt[3]{9}$ .
16.  $\sqrt[4]{5}$ ;  $3\sqrt[4]{5}$ .
17.  $3\sqrt{2}$ ;  $8\sqrt{2}$ ;  $4\sqrt{2}$ .
18.  $21\sqrt[3]{2}$ ;  $6\sqrt[3]{2}$ ;  $6\sqrt[3]{2}$ .
19.  $3\sqrt{3}$ ;  $8\sqrt{3}$ ;  $18\sqrt{3}$ .
20.  $4\sqrt[3]{2}$ ;  $7\sqrt[3]{2}$ ;  $2\sqrt[3]{2}$ .
21.  $3\sqrt[4]{2}$ ;  $6\sqrt[4]{2}$ ;  $6\sqrt[4]{2}$ .
22.  $2\sqrt{\frac{1}{8}}$ ;  $10\sqrt{\frac{1}{8}}$ .
23.  $\frac{1}{2}\sqrt{\frac{2}{5}}$ ;  $\frac{1}{8}\sqrt{\frac{2}{5}}$ .
24.  $\frac{1}{7}\sqrt{\frac{2}{3}}$ ;  $\frac{1}{11}\sqrt{\frac{2}{3}}$ .
25.  $\frac{1}{4}\sqrt{0.2}$ ;  $\sqrt{0.2}$ .
26.  $10\sqrt{\frac{1}{5}}$ ;  $3\sqrt{\frac{1}{5}}$ ;  $100\sqrt{\frac{1}{5}}$ ;  $21\sqrt{\frac{1}{5}}$ ;  $\frac{5}{2}\sqrt{\frac{1}{5}}$ ;  $\frac{3}{4}\sqrt{\frac{1}{5}}$ .
27.  $2\sqrt[3]{\frac{1}{8}}$ ;  $\frac{3}{2}\sqrt[3]{\frac{1}{8}}$ ;  $\frac{6}{7}\sqrt[3]{\frac{1}{8}}$ ;  $\sqrt[3]{5}\sqrt[3]{\frac{1}{8}}$ ;  $2\sqrt[3]{2}\sqrt[3]{\frac{1}{8}}$ .
28.  $\frac{1}{2}\sqrt[5]{6}$ ;  $2\sqrt[5]{6}$ ;  $\frac{1}{4}\sqrt[5]{6}$ ;  $\frac{1}{5}\sqrt[5]{3}\sqrt[5]{6}$ ;  $\frac{1}{8}\sqrt[5]{6}$ .
29.  $\frac{1}{2}\sqrt{5}$ ;  $\frac{1}{3}\sqrt{5}$ .
30.  $\frac{1}{2}\sqrt{\frac{1}{5}}$ ;  $\frac{1}{8}\sqrt{\frac{1}{5}}$ .
31.  $\frac{1}{2}\sqrt{3}$ ;  $\frac{4}{9}\sqrt{3}$ .
32.  $\frac{1}{5}\sqrt[3]{7}$ ;  $\frac{1}{4}\sqrt[3]{7}$ .
33.  $a\sqrt[6]{ab}$ ;  $a^2b\sqrt[6]{ab}$ .
34.  $3m\sqrt[3]{m}$ ;  $2m^2\sqrt[3]{m}$ .
35.  $x\sqrt[3]{x(x-3)}$ ;  $\frac{1}{x}\sqrt[3]{x(x-3)}$ .
36.  $\frac{a^2}{b^2}\sqrt{\frac{c}{b}}$ ;  $\frac{ac}{x}\sqrt{\frac{c}{b}}$ ;  $\frac{ax}{y}\sqrt{\frac{c}{b}}$ .

37.  $\frac{a}{b}\sqrt{\frac{ax}{b}}; \frac{x^2}{b^2}\sqrt{\frac{ax}{b}}; \frac{x}{a}\sqrt{\frac{ax}{b}}.$
38.  $\frac{1}{a}\sqrt[4]{a}; \frac{b}{a}\sqrt[4]{a}; \sqrt[4]{a}; \frac{1}{cx^2}\sqrt[4]{a}.$
39.  $\frac{1}{a}\sqrt{a^2-1}; \frac{1}{a+1}\sqrt{a^2-1}.$
40.  $\frac{1}{b}\sqrt{b(1-ab)}; \frac{d}{c}\sqrt{b(1-ab)}.$
41.  $(a-b)^2\sqrt{\frac{1}{a-b}}; (a^2+b^2)\sqrt{\frac{1}{a-b}}; \text{ or, } (a-b)\sqrt{a-b};$   
 $\frac{a^2+b^2}{a-b}\sqrt{a-b}.$
42.  $(x-b^2)\sqrt[3]{\frac{1}{x-b^2}}; (y-z)\sqrt[3]{\frac{1}{x-b^2}}; y\sqrt[3]{5y^2}\sqrt[3]{\frac{1}{x-b^2}}.$
43.  $\frac{m}{1-x}\sqrt{m}; \frac{2(1-x)}{cn}\sqrt{m}.$
44.  $4\sqrt{2}.$       48.  $3\sqrt{6}.$       52.  $-\sqrt{2}.$
45.  $\sqrt{147}.$       49.  $4\sqrt{5}.$       53.  $\sqrt{3} + \sqrt{19}.$
46.  $8\sqrt{2}.$       50.  $\sqrt{63}=3\sqrt{7}.$       54.  $\sqrt{8} + \sqrt{12}.$
47.  $10\sqrt{5}.$       51.  $3\sqrt{11}.$       55.  $\sqrt{10} + \sqrt{8} + \sqrt{6}.$

## Ex. 44. Page 152.

1.  $5\sqrt{3}.$       5.  $2\sqrt{11}.$       9.  $4\sqrt{2}.$       13.  $\frac{7}{2}\sqrt[3]{5}.$
2.  $10\sqrt{7}.$       6.  $6\sqrt{3}.$       10.  $6\sqrt[3]{4}.$       14.  $-4\sqrt[4]{3}.$
3.  $13\sqrt{10}.$       7.  $3\sqrt[3]{3}.$       11.  $3\sqrt{2}.$       15.  $11\sqrt{7}.$
4.  $4\sqrt{2}.$       8.  $7\sqrt{13}.$       12.  $5\sqrt[5]{2}.$       16.  $-\sqrt{2}.$
17.  $12\sqrt{3}.$       18.  $129\sqrt{5}.$       19.  $19\sqrt{5} - 23\sqrt{3}.$

- |  |                               |
|--|-------------------------------|
| 20. $35\sqrt{6}$ .                                 | 33. $5\sqrt{b}$ .             |
| 21. $6\sqrt{5}$ .                                  | 34. $(c+5)\sqrt[3]{c}$ .      |
| 22. $4\sqrt{13}-3\sqrt{11}$ .                      | 35. $(a-b+2)\sqrt[5]{b}$ .    |
| 23. $-\frac{11}{2}\sqrt{2}-1\frac{1}{2}\sqrt{5}$ . | 36. $(a-b)^2\sqrt{x}$ .       |
| 24. $4\sqrt[3]{5}-11\frac{1}{2}\sqrt[3]{4}$ .      | 37. $(x+y)^2\sqrt{z}$ .       |
| 25. $\frac{1}{3}+\frac{1}{6}\sqrt{5}$ .            | 38. $2a(b-1)\sqrt{c}$ .       |
| 26. $\frac{1}{2}\sqrt{2}-\frac{1}{2}\sqrt[3]{2}$ . | 39. $(7a-2)\sqrt[4]{a}$ .     |
| 27. $2\sqrt{15}$ .                                 | 40. $(3m+1)\sqrt[3]{m}$ .     |
| 28. $6\sqrt{6}-2\sqrt{11}$ .                       | 41. $-(7+5a)\sqrt{2a}$ .      |
| 29. $4\frac{4}{5}\sqrt{6}-\sqrt[3]{14}$ .          | 42. $10ab\sqrt{7ab}$ .        |
| 30. $2\sqrt{a}$ .                                  | 43. $4mn\sqrt{5m}$ .          |
| 31. $-7\frac{1}{2}\sqrt[3]{a^3}$ .                 | 44. $(2ab-6)^2\sqrt{2b}$ .    |
| 32. $(a+b-3)\sqrt{a}$ .                            | 45. $2a^2(2+5b)\sqrt[3]{b}$ . |

## Ex. 45. Page 153.

- |        |                      |                                 |                       |
|--------|----------------------|---------------------------------|-----------------------|
| 1. 9.  | 12. -7.              | 23. $2\sqrt[4]{3}$ .            | 34. $-5\sqrt[3]{6}$ . |
| 2. 10. | 13. 11.              | 24. $3\sqrt[5]{2}$ .            | 35. $\sqrt{21}$ .     |
| 3. 6.  | 14. -2.              | 25. $-9\sqrt[3]{5}$ .           | 36. $\sqrt{7}$ .      |
| 4. 3.  | 15. 27.              | 26. 8.                          | 37. 8.                |
| 5. 4.  | 16. 72.              | 27. 60.                         | 38. 5.                |
| 6. 8.  | 17. 25.              | 28. $\frac{3}{4}$ .             | 39. 7.                |
| 7. 3.  | 18. 729.             | 29. $-\frac{2}{3}\sqrt[3]{2}$ . | 40. 4.                |
| 8. 6.  | 19. 60.              | 30. 24.                         | 41. 4.                |
| 9. 2.  | 20. 14.              | 31. 10.                         | 42. 5.                |
| 10. 3. | 21. $3\sqrt{2}$ .    | 32. 4.                          | 43. $\sqrt{5}-4$ .    |
| 11. 8. | 22. $3\sqrt[3]{2}$ . | 33. $-237\frac{1}{4}$ .         | 44. $2(5-\sqrt{2})$ . |

45.  $-(15 + 19\sqrt{3})$ .      46.  $3(12 - 5\sqrt{6})$ .  
 47.  $6\sqrt{2} - 3\sqrt{15} + 8\sqrt{3} - 6\sqrt{10}$ .  
 48.  $\sqrt{14} - \sqrt{6} + \sqrt{35} - \sqrt{15}$ .  
 49.  $8 - 8\sqrt[3]{12} + \sqrt[3]{18}$ .      50. 30.  
 51.  $54 - 7\sqrt{35} - 26\sqrt{21} - 7\sqrt{15}$ .  
 52.  $171 - 36\sqrt{6}$ .      53.  $86 + 23\frac{1}{2}\sqrt{6}$ .      54.  $140 + 10\sqrt{6}$ .  
 55.  $\sqrt[3]{12} - 2\sqrt[3]{3} + 2\sqrt[3]{18} + 3\sqrt[3]{2} + 4\sqrt[3]{9} - 3\sqrt[3]{4} - 1$ .  
 56.  $156 - 24\sqrt[3]{4}$ .      58.  $4 - \frac{3}{2}\sqrt[3]{3} + 25\sqrt[3]{9}$ .  
 57.  $340 + 51\sqrt[3]{18}$ .      59.  $-\sqrt{2}$ .  
 60.  $176 + 66\sqrt{5}$ .  
 61.  $110 + 24\sqrt{10} - 12\sqrt{6} - 16\sqrt{15}$ .  
 62.  $33 - 18\sqrt{2}$ .      65. 5887.  
 63. 125.      66.  $75\frac{3}{4} + 7\frac{1}{2}\sqrt{2}$ .  
 64. 243.      67.  $9x^5 - 12x^4 + 4x^3$ .  
 68.  $a^2y + 2aby^2 + b^2y^3$ .  
 69.  $ab^3m^4 - 2a^2b^2m^3 + a^3bm^2$ .  
 70.  $48p^2z^5 - 12pqz^3\sqrt{6} + \frac{3}{2}q^2z$ .  
 71.  $\sqrt[5]{2000}$ .      78. 15.      85.  $\sqrt[15]{144}$ .  
 72.  $5\sqrt{30}$ .      79.  $\sqrt[10]{13,824}$ .      86.  $\sqrt[20]{\frac{5^4 \times 7}{3^6}}$ .  
 73.  $2\sqrt{10}$ .      80.  $6\sqrt[3]{9}$ .      87.  $\sqrt[20]{\frac{9}{2^{15}}} = \frac{\sqrt[10]{3}}{\sqrt[4]{8}}$ .  
 74. 6.      81.  $\sqrt[20]{81 \times 42^5}$ .      88.  $\sqrt[36]{\frac{2}{3 \times 7^4}}$ .  
 75. 4.      82.  $\sqrt[15]{2^8}$ .      89.  $\sqrt[30]{\frac{2 \cdot 5 \cdot 6}{3}}$ .  
 76.  $2\sqrt[3]{25}$ .      83.  $7\sqrt[14]{7^3 \times 8^7}$ .  
 77. 2.      84.  $6\sqrt[30]{6 \times 7^{21}}$ .

## Ex. 46. Page 156.

- |   |  |   |                             |                     |                       |
|---|--|---|-----------------------------|---------------------|-----------------------|
| 1. 9.                                     | 4. 5.                                  | 7. 2.   | 10. 4.                      | 13. $\frac{7}{8}$ . | 16. $1\frac{1}{8}$ .  |
| 2. 3.                                     | 5. 6.                                  | 8. 3.   | 11. $\frac{3}{8}$ .         | 14. $\frac{3}{8}$ . | 17. $1\frac{1}{8}$ .  |
| 3. 2.                                     | 6.                                     | 9. 5.   | 12. $\frac{5}{8}\sqrt{2}$ . | 15. $\frac{7}{8}$ . | 18. $1\frac{1}{17}$ . |
| 19. $\sqrt{2} + 5\sqrt{6}$ .              |  | 23. $2\sqrt{42} - 3\sqrt{14} + 5\sqrt{10}$ .          |                             |                     |                       |
| 20. $7\sqrt{3} - 3\sqrt{5}$ .             |  | 24. $2\sqrt[3]{25} - 6\sqrt[3]{4} + 5\sqrt[3]{36}$ .  |                             |                     |                       |
| 21. $4\sqrt{35} + 8\sqrt{14}$ .           |  | 25. $5\sqrt[3]{75} + 3\sqrt[3]{18} - 4\sqrt[3]{36}$ . |                             |                     |                       |
| 22. $5\sqrt{7} - 4\sqrt{6} + 2\sqrt{5}$ . |  | 26. $3 - 2\sqrt{2}$ .                                 | 27. $\sqrt{3}$ .            |                     |                       |
| 28. $\sqrt{x}$ .                          | 43. $\frac{3}{2y}$ .                   | 58. 0.2.  |                             |                     |                       |
| 29. $\sqrt{2}$ .                          | 44. $\frac{3a}{2y^2}\sqrt[4]{12y^2}$ . | 59. 0.3.  |                             |                     |                       |
| 30. $\sqrt{2m}$ .                         | 45. $\frac{2}{3}\sqrt{5m}$ .           | 60. $\frac{1}{15}\sqrt{10}$ .                         |                             |                     |                       |
| 31. $\frac{1}{2}\sqrt{10}$ .              | 46. $\sqrt{a} + \sqrt{b}$ .            | 61. $\frac{1}{3}\sqrt{10}$ .                          |                             |                     |                       |
| 32. $a$ .                                 | 47. $1 + \sqrt{a}$ .                   | 62. $\frac{1}{2}\sqrt[5]{16}$ .                       |                             |                     |                       |
| 33. $a^2\sqrt{3}$ .                       | 48. $\sqrt{b} + 1$ .                   | 63. $\sqrt{3}$ .                                      |                             |                     |                       |
| 34. $2c^2$ .                              | 49. $x - y$ .                          | 64. 7.  |                             |                     |                       |
| 35. $\frac{a}{b}$ .                       | 50. $ax\sqrt{x} - b\sqrt{y}$ .         | 65. 5.  |                             |                     |                       |
| 36. $\frac{1}{2a}\sqrt{2}$ .              | 51. $cy\sqrt[3]{y^2} - d\sqrt[3]{z}$ . | 66. $2\sqrt{3}$ .                                     |                             |                     |                       |
| 37. $3a^2$ .                              | 52. 3.                                 | 67. $\frac{1}{2}\sqrt{6}$ .                           |                             |                     |                       |
| 38. $2n\sqrt[3]{2}$ .                     | 53. $\sqrt[6]{3}$ .                    | 68. $\frac{2}{3}\sqrt{2}$ .                           |                             |                     |                       |
| 39. $7x\sqrt[3]{x^2}$ .                   | 54. $\sqrt[6]{7}$ .                    | 69. $\frac{1}{3}\sqrt{7}$ .                           |                             |                     |                       |
| 40. $2y$ .                                | 55. $\sqrt[6]{24}$ .                   | 70. $\frac{1}{4}\sqrt{210}$ .                         |                             |                     |                       |
| 41. $\frac{1}{x^2}\sqrt[3]{ax^2}$ .       | 56. $\sqrt[6]{\frac{2}{3}}$ .          | 71. $9\sqrt{3} + 15$ .                                |                             |                     |                       |
| 42. $5a\sqrt[4]{a}$ .                     | 57. 0.6.                               | 72. $6\sqrt{2} + 8$ .                                 |                             |                     |                       |
|   |  | 73. $\frac{1}{a}\sqrt[6]{a^5}$ .                      |                             |                     |                       |

74.  $\sqrt[15]{b}$ . 75.  $\frac{1}{m} \sqrt[12]{m^{11}}$ . 76.  $\sqrt[18]{x^{11}}$ .
77.  $\frac{1}{x} \sqrt[6]{\frac{4}{x}} = \frac{1}{x^2} \sqrt[6]{4x^5}$ .
78.  $\sqrt[35]{\frac{y^{13}}{32}}$ . 82.  $a\sqrt{a}$ . 87.  $\frac{1}{2x^2y^3} \sqrt[4]{8x^3y^2}$ .
79.  $\frac{1}{6z} \sqrt{2}$ . 83.  $b^2\sqrt{b}$ . 88.  $\frac{2a}{5x} \sqrt[3]{\frac{5a^2}{x}}$ .
80.  $\frac{1}{a} \sqrt[7]{a^{68}b^{30}}$ . 84.  $z^4\sqrt{z}$ . 89.  $3\sqrt{\frac{x+y}{x-y}}$ .
81.  $\sqrt[42]{\frac{a^5}{c^{10}}}$ . 85.  $\frac{y^3}{2} \sqrt{6y}$ . 90.  $\frac{1}{2} \sqrt[4]{m^3} + \frac{2}{3} \sqrt[4]{mn^3}$ .
91.  $\frac{1}{8}$ . 101.  $\frac{1}{a}$ . 108.  $a^4$ .
92. 4. 102.  $\frac{n^3}{m^2 \sqrt[3]{m}}$ . 109.  $\sqrt[3]{\frac{1}{m^7}}$ .
93.  $\frac{5}{2}$ . 103.  $\frac{b^2\sqrt{b}}{a\sqrt{a}}$ . 110.  $\sqrt[6]{a^5}$ .
94.  $\frac{1}{\sqrt{6}}$ . 104.  $a$ . 111.  $x^2 \times \sqrt[10]{x^7}$ .
95. 17. 105.  $m^{15}\sqrt{m}$ . 112.  $\frac{1}{\sqrt[15]{a}}$ .
96.  $\frac{1}{8}$ . 106.  $\frac{1}{x^5 \sqrt[6]{x}}$ . 113.  $4\sqrt[3]{4}$ .
97. 4. 107.  $\sqrt[10]{\frac{1}{a^{11}}}$ . 114.  $24\sqrt{3}$ .
98.  $\frac{1}{8}$ . 115.  $\sqrt[3]{9}$ .
99.  $\frac{1}{\sqrt{a}}$ .
100.  $\frac{1}{m^2}$ .

Ex. 47. Page 159.

1.  $a^3$ . 3.  $n^3$ . 5.  $y^2$ . 7.  $\sqrt{m^{15}}$ .
2.  $m^2$ . 4.  $x^3$ . 6.  $ab^2$ . 8.  $n^2\sqrt[3]{n^2}$ .

- |   |  |   |
|---|--|---|
| 9. $y\sqrt[4]{y}$ .   | 14. $3a^3$ .                             | 19. $(x-y)\sqrt[5]{(x-y)^3}$            |
| 10. $b^3c\sqrt[3]{b^3c}$ .  | 15. $a^3c$ .                             | 20. $(a^2 - b^2)^2$ .                   |
| 11. $xz^2\sqrt[3]{xz^2}$ .  | 16. $2a^4\sqrt[4]{a}$ .                  | 21. $\sqrt[4]{(1+a^2)^3}$ .             |
| 12. $a^3\sqrt[3]{4a^3}$ .   | 17. $a^2x^3$ .                           | 22. $(x^3 - y^3)^{2n}$ .                |
| 13. $1\frac{2}{3}$ .  | 18. $(x+y)^6$ .                          |   |
| 23. $\sqrt[3]{a^4b^2} + m^3 + \sqrt{ax^2} - 2\sqrt[6]{a^4b^3m^9} + 2\sqrt[12]{a^{11}b^4x^9} - 2\sqrt[4]{m^6ax^3}$ . |  |   |
| 24. $\sqrt[3]{a^4} + \sqrt[3]{b^2} - 2\sqrt[3]{a^2b^2}$ .   |  |   |
| 25. $\sqrt[3]{m^3} + \sqrt[3]{n^2} + \sqrt[3]{p^4} - 2\sqrt[3]{m^2n^2} + 2\sqrt[3]{m^2p^2} - 2\sqrt[3]{n^2p^2}$ .   |  |   |
| 26. $2a^3b$ .   | 39. $v^3 = \sqrt[3]{b^3}$ .              | 52. $\sqrt[12]{m^5}$ .                  |
| 27. $-xy^3$ .   | 40. $(x^4)^3 = \sqrt[3]{x^4} = x^{12}$ . | 53. $\sqrt{\sqrt[3]{125}} = \sqrt{5}$ . |
| 28. $(x^2 - y^2)^3$ .   | 41. $\sqrt[4]{x^3}$ .                    | 54. $\sqrt{3}$ .                        |
| 29. $-(a^2 - b^2)^3$ .  | 42. $\sqrt[3]{y^5} = y^{20}$ .           | 55. $x - y$ .                           |
| 30. 5.  | 43. $\sqrt[3]{a}$ .                      | 56. $(a - b)^2$ .                       |
| 31. 16.   | 44. $\sqrt[6]{b}$ .                      | 57. $(m + n)^2$ .                       |
| 32. 127.  | 45. $\sqrt[12]{c}$ .                     | 58. $\sqrt[3]{(3a - 2b)^2}$ .           |
| 33. $-131$ .  | 46. $\sqrt[30]{2m}$ .                    | 59. $(1 - x^2)^4$ .                     |
| 34. $9\sqrt[3]{9}$ .  | 47. $\sqrt[8]{3a^2}$ .                   | 60. $\sqrt[3]{2x^2}$ .                  |
| 35. $-4\sqrt[4]{2}$ .   | 48. $\sqrt[3]{a}$ .                      | 61. $a^3\sqrt[3]{2}$ .                  |
| 36. $-13^2 = 169$ .   | 49. $\sqrt{m}$ .                         | 62. $m\sqrt[3]{n^2}$ .                  |
| 37. 6.  | 50. $\sqrt[3]{y}$ .                      | 63. $m\sqrt[13]{6}$ .                   |
| 38. $\sqrt{n}$ .  | 51. $\sqrt[3]{a^3}$ .                    | 64. $2a^3\sqrt[3]{3}$ .                 |

## Ex. 48. Page 160.

1.  $\frac{2(3 + \sqrt{6})}{3}$ .

6.  $4 - \sqrt{15}$ .

2.  $\frac{21(4 + \sqrt{6})}{10}$ .

7.  $\frac{8 + \sqrt{55}}{3}$ .

3.  $4(5\sqrt{3} - 3\sqrt{5})$ .

8.  $\frac{23 - 2\sqrt{130}}{3}$ .

4.  $\frac{3(8\sqrt{7} - 7\sqrt{2})}{2}$ .

9.  $\frac{11 + 3\sqrt{14}}{5}$ .

5.  $40(3\sqrt{2} + \sqrt{3})$ .

10.  $4 + \sqrt{15}$ .

11.  $-\frac{12 + 9\sqrt{2} + 20\sqrt{3} + 15\sqrt{6}}{2}$ .

12.  $\frac{3 - \sqrt{3} - 12\sqrt{5} + 4\sqrt{15}}{12}$ .

13.  $\frac{7(4\sqrt{5} - 5) + 15(\sqrt{10} - 4\sqrt{2})}{55}$ .

14.  $\frac{5(2\sqrt{3} + \sqrt{6}) - 6(1 + \sqrt{2})}{57}$ .

15.  $-\frac{7(2 + \sqrt{11}) + 3(2\sqrt{7} + \sqrt{77})}{42}$ .

16.  $\frac{18(120\sqrt{6} + 205\sqrt{3} - 219\sqrt{2} - 308)}{529}$ .

17.  $\frac{\sqrt{2} + \sqrt{6}}{2}$ . 18.  $\sqrt{3} + \sqrt{2}$ . 19.  $2\sqrt{2} + \sqrt{5} - \sqrt{10} - 2$ .

20.  $\frac{195 - 75\sqrt{3} - 187\sqrt{2} + 137\sqrt{6}}{94}$ .

21.  $\frac{1}{2}(19\sqrt{2} + 16\sqrt{3} - 10\sqrt{5} - 10\sqrt{6} + 8\sqrt{10} + 8\sqrt{15} - 5\sqrt{30} - 22)$ .



T 128.85.824



HARVARD  
COLLEGE  
LIBRARY

THE GIFT OF

*Miss Ellen Lang Wentworth  
of Exeter, New Hampshire*



## Ex. 50. Page 163.

1. 2.	26. 2.	51. 4.	75. $8; 4\frac{4}{7}$ .
2. 3.	27. $-\frac{5}{7}$ .	52. 4.	76. $4; -\frac{4}{7}$ .
3. $2\frac{5}{8}$ .	28. $-3$ .	53. $\frac{a}{a+2\sqrt{a}}$ .	77. $\frac{1}{2}; -4\frac{1}{2}$ .
4. $12\frac{1}{4}$ .	29. 1.	54. $\sqrt{3}; 0$ .	78. $\frac{5}{4}; 55\frac{1}{4}$ .
5. $\frac{1}{8}$ .	30. $\frac{1}{4}$ .	55. $\frac{1}{2}\sqrt{3}$ .	79. $4; -\frac{8}{9}$ .
6. $\frac{1}{4}$ .	31. 49.	56. 3.	80. $-4; -7$ .
7. $\frac{4}{9}$ .	32. 5.	57. $6; 2$ .	81. $3; \frac{9}{10}$ .
8. $-\frac{1}{8}$ .	33. 23.	58. $7; 3$ .	82. $2; -16\frac{1}{2}$ .
9. $\frac{1}{20}$ .	34. 8.	59. $4; -25$ .	83. $1; -\frac{11}{2}$ .
10. 2.	35. 6.	60. $196; 49$ .	84. $3; -3\frac{5}{7}$ .
11. $-2$ .	36. 5.	61. $1\frac{8}{15}; \frac{2}{3}$ .	85. $7; -6$ .
12. $-\frac{1}{8}$ .	37. 49.	62. $2; \frac{1}{2}$ .	86. $\frac{1}{2}; -\frac{5}{7}$ .
13. $6\frac{2}{3}$ .	38. 4.	63. $5; -2$ .	87. $1; -\frac{1}{8}$ .
14. $-2\frac{1}{2}$ .	39. 4.	64. $6\frac{9}{10}; 6$ .	88. $5; -4\frac{1}{18}$ .
15. $-6$ .	40. $8\frac{1}{2}$ .	65. $7; 0$ .	89. $3; -\frac{2}{3}$ .
16. 2.	41. 60.	66. $2; -\frac{8}{10}$ .	90. $\pm 2$ .
17. 5.	42. $4\frac{1}{2}$ .	67. $6; 1\frac{4}{5}$ .	91. $\pm \frac{1}{2}$ .
18. $-5$ .	43. $\frac{1}{9}$ .	68. $5; \frac{5}{8}$ .	92. $\frac{2}{3}; \frac{1}{5}$ .
19. $1\frac{2}{3}$ .	44. $-\frac{1}{4}$ .	69. $2; -\frac{10}{11}$ .	93. $4; \frac{1}{2}$ .
20. $-5$ .	45. $\frac{1}{8}$ .	70. $9; 1$ .	94. $\pm 2$ .
21. 1.	46. 25.	71. $4; \frac{8}{9}$ .	95. $\pm \frac{2}{3}$ .
22. 4.	47. $\frac{3}{4}$ .	72. $3; -\frac{11}{8}$ .	96. $4; -\frac{1}{4}$ .
23. $\frac{7}{9}$ .	48. $\frac{1}{8}$ .	73. $5; 1$ .	97. $\frac{b}{a}; -1$ .
24. $2\frac{1}{2}$ .	49. $\frac{1}{8}$ .	74. $\frac{1}{2}; -\frac{9}{10}$ .	98. $\frac{4a}{3}; \frac{a}{3}$ .
25. $\frac{1}{2}$ .	50. $200\frac{1}{2}$ .		

99.  $\frac{1}{2a}(1 \pm \sqrt{1+4b^3})$ .      102.  $2a; -\frac{2a}{3}$   
 100.  $\frac{1}{2}(\sqrt{a} \pm \sqrt{b})$ .      103.  $4a; -2a$ .  
 101.  $\frac{a}{4}(1 \pm \sqrt{7})$ .      104.  $\frac{a}{3}; -\frac{a+2b}{3}$ .  
 105.  $\frac{a(2 \pm \sqrt{2})}{2}$ .      110.  $\pm \frac{a}{2}\sqrt{\frac{a^2-4}{a^3-1}}$ .      113.  $\frac{a}{2b^2}; -\frac{9a}{2b^2}$ .  
 106.  $\pm \frac{2a}{\sqrt{5}}$ .      111.  $0; \frac{9a}{16}$ .      114.  $0; a$ .  
 107.  $b \pm \sqrt{b^2-ab}$ .      115.  $\pm \sqrt{ab}$ .  
 108.  $\pm \sqrt{n}$ .      112.  $\pm a\sqrt{\frac{a}{a-1}}$ .      116.  $\frac{a}{1+\sqrt[3]{a^2}}$ .  
 109.  $\pm 2b$ .

$$117. \frac{a+b}{2} \pm \frac{a-b}{4} \sqrt{2}.$$

118.  $\pm 2a$  are evidently the two roots.

119. The roots are evidently  $a$  and  $b$ .

120.  $\frac{a-b}{2} \pm \frac{(a+b)c}{2\sqrt{c^2+4}}$ .      122.  $\frac{a+b}{2} \pm \frac{a-b}{2c} \sqrt{c^2+4}$ .  
 121.  $\frac{a+b}{2} \pm \frac{(a-b)c}{2\sqrt{c^2-4}}$ .      123.  $x=6a$ , or  $-\frac{2a}{3}$ .

**Ex. 51. Page 169.**

- |                 |                 |                  |
|-----------------|-----------------|------------------|
| 1. \$3900.48.   | 5. \$1377.01.   | 9. \$124,225.88. |
| 2. \$13,823.13. | 6. \$8,646,290. | 10. \$5,643.20.  |
| 3. \$4855.87.   | 7. \$6250.      | 11. \$3,129.29.  |
| 4. \$35,705.48. | 8. \$368,429.   | 12. \$606.77.    |
| 13. \$1459.52.  |                 |                  |

14. First, \$2368.39; second, \$2377.52; third, \$2382.18; fourth, \$2385.34. 21. 8 yrs.
15. \$7800. 18. \$955.025. 22. 11 yrs.
16. \$1400. 19. \$831.04. 23. About  $7\frac{1}{2}$  yrs.
17. \$4315. 20. \$1614.97. 24. About  $54\frac{1}{2}$  yrs.
25. First, 23.45 yrs.; second, 20.15 yrs.; third, 17.67 yrs.; fourth,  $14\frac{1}{2}$  yrs., nearly. 38. 4.651%.
26. 4%. 32. \$3916.80. 39. \$2812.83.
27.  $4\frac{1}{2}$ %. 33. 21.64%. 40. \$51,280
28.  $4\frac{1}{2}$ %. 34. \$110.28. and \$52,700.
29. 3%. 35. 3%. 41. \$5097.97
30. 6%. 36. 67.826 gals. and \$5490.97.
31. \$1260.12. 37. 74,272.88<sup>cbm</sup>. 42. 100 yrs.
43. The \$35,000 of B, discounted at 5%, is reduced to \$30,234.31; the \$33,000 of C to \$31,453.52. Then C has offered \$1453.52 more than A, and \$1219.21 more than B.
44. \$10,297.16. 50. \$66,868. 57. \$16,431.95.
45. 20 yrs. 51. \$76,080. 58. 17 yrs.
46. 24 yrs. 52. 1979.62<sup>m</sup>. 59. \$2200.50.
47. \$23,569.63. 53. 13.58 yrs. 60. \$105,078.50.
48. \$12,144.28. 54. 21.26 yrs. 61. \$5000.
49. 1,111,282 inhabitants. 55. 11 yrs. 62.  $4\frac{1}{2}$ %.
56. \$12,500.

## Ex. 52. Page 176.

1. 4 : 1. 4. 1 : 10. 7. 11 : 9. 10. 5 : 9.
2. 3 : 2. 5. 9 : 1. 8. 13 : 16. 11. 3 : 10.
3. 1 : 7. 6. 7 : 8 : 9 : 10. 9. 6 : 5. 12. 9 : 16.
14.  $\frac{m}{t} : \frac{n}{p}$ ; or  $mp : nt$ ; 14 : 15. 13. 8 : 27.
15.  $\frac{18}{27} = \frac{59}{75}$ . 17. 150. 19.  $9\frac{1}{2}$ . 21.  $11\frac{1}{2}$ .
16.  $\frac{285}{888} = \frac{195}{592}$ . 18.  $3\frac{1}{2}$ . 20.  $1\frac{1}{2}$ . 23. 14.

- |                               |                                      |                            |                       |
|-------------------------------|--------------------------------------|----------------------------|-----------------------|
| 24. 18.                       | 30. $4\frac{1}{8}$ .                 | 36. $\frac{a^2(a-b)}{c}$ . | 40. $4\frac{3}{8}$ .  |
| 25. $6\frac{1}{8}$ .          | 31. $2\frac{3}{8}$ .                 |                            | 41. $45\frac{1}{8}$ . |
| 26. $1\frac{3}{8}$ .          | 32. $\frac{4}{8}$ .                  | 37. $\frac{1-b}{a}$ .      |                       |
| 27. $3n$ .                    | 33. 1.                               |                            | 42. $\frac{np}{m}$ .  |
| 28. $\frac{1}{8}$ qt.         | 34. $(p+q)^2$ .                      | 38. 1.                     |                       |
| 29. $\frac{6bpq}{a}$ .        | 35. $\frac{1}{m-n}$ .                | 39. 12.                    | 43. $\frac{a}{bc}$ .  |
| 44. $\frac{m(m-n)}{n(m+n)}$ . | 51. $\frac{n^2}{m}$ .                |                            | 56. 9.                |
| 45. $\frac{1-n}{m}$ .         | 52. $\frac{p-q}{r(p+q)}$ .           |                            | 57. 6.                |
| 46. $\frac{a^2+b^2}{ab}$ .    | 53. $\frac{4p^2}{(m^2-mp+p^2)^2}$ .  |                            | 58. 10.               |
| 47. 4.                        | 54. $\frac{(l-m)^2}{(l+m)^2}$ .      |                            | 59. 5.                |
| 48. 9.                        | 55. $\frac{4(a-b)^4}{9b^4(a+b)^2}$ . |                            | 60. 7.                |
| 49. 25.                       |                                      |                            | 61. $m$ .             |
| 50. $10\frac{1}{8}$ .         |                                      |                            | 62. $\frac{q}{2p}$ .  |
|                               |                                      |                            | 63. 1.                |
|                               |                                      |                            | 64. $c+d$ .           |

## Ex. 53. Page 179.

- |   |                                   |  |
|---|-----------------------------------|--|
| 1. $l=55$ ,<br>$s=403$ .                      | 6. $l=28$ ,<br>$s=364$ .          | 11. $d=4$ ,<br>$s=5100$ .                      |
| 2. $l=38$ ,<br>$s=258$ .                      | 7. $l=73n$ ,<br>$s=1368n$ .       | 12. $d=\frac{3}{4}$ ,<br>$s=2757\frac{1}{4}$ . |
| 3. $l=174\frac{1}{2}$ ,<br>$s=2625$ .         | 8. $l=2m+52n$ ,<br>$s=28m+364n$ . | 13. $d=-7$ ,<br>$s=2124$ .                     |
| 4. $l=20.9$ ,<br>$s=2190$ .                   | 9. $l=1+30m$ ,<br>$s=11+165m$ .   | 14. $d=-0.3$ ,<br>$s=63$ .                     |
| 5. $l=10\frac{1}{4}$ ,<br>$s=83\frac{1}{4}$ . | 10. $d=11$ ,<br>$s=884$ .         | 15. $d=7a^2$ ,<br>$s=1575a^3$ .                |

## Ex. 46. Page 156.

- |   |  |                       |   |                     |                       |
|---|--|-----------------------|---|---------------------|-----------------------|
| 1. 9.                                     | 4. 5.                                  | 7. 2.                 | 10. 4.  | 13. $\frac{7}{8}$ . | 16. $1\frac{1}{2}$ .  |
| 2. 3.                                     | 5. 6.                                  | 8. 3.                 | 11. $\frac{3}{4}$ .                                   | 14. $\frac{3}{8}$ . | 17. $1\frac{1}{2}$ .  |
| 3. 2.                                     | 6.                                     | 9. 5.                 | 12. $\frac{5}{8}\sqrt{2}$ .                           | 15. $\frac{7}{8}$ . | 18. $1\frac{2}{17}$ . |
| 19. $\sqrt{2} + 5\sqrt{6}$ .              |  |                       | 23. $2\sqrt{42} - 3\sqrt{14} + 5\sqrt{10}$ .          |                     |                       |
| 20. $7\sqrt{3} - 3\sqrt{5}$ .             |  |                       | 24. $2\sqrt[3]{25} - 6\sqrt[3]{4} + 5\sqrt[3]{36}$ .  |                     |                       |
| 21. $4\sqrt{35} + 8\sqrt{14}$ .           |  |                       | 25. $5\sqrt[3]{75} + 3\sqrt[3]{18} - 4\sqrt[3]{36}$ . |                     |                       |
| 22. $5\sqrt{7} - 4\sqrt{6} + 2\sqrt{5}$ . |  | 26. $3 - 2\sqrt{2}$ . | 27. $\sqrt{3}$ .                                      |                     |                       |
| 28. $\sqrt{x}$ .                          | 43. $\frac{3}{2y}$ .                   |                       | 58. 0.2.  |                     |                       |
| 29. $\sqrt{2}$ .                          |  |                       | 59. 0.3.  |                     |                       |
| 30. $\sqrt{2m}$ .                         | 44. $\frac{3a}{2y^2}\sqrt[4]{12y^2}$ . |                       | 60. $\frac{1}{15}\sqrt{10}$ .                         |                     |                       |
| 31. $\frac{1}{2}\sqrt{10}$ .              | 45. $\frac{2}{3}\sqrt{5m}$ .           |                       | 61. $\frac{1}{8}\sqrt{10}$ .                          |                     |                       |
| 32. $a$ .                                 | 46. $\sqrt{a} + \sqrt{b}$ .            |                       | 62. $\frac{1}{2}\sqrt[5]{16}$ .                       |                     |                       |
| 33. $a^2\sqrt{3}$ .                       | 47. $1 + \sqrt{a}$ .                   |                       | 63. $\sqrt{3}$ .                                      |                     |                       |
| 34. $2c^2$ .                              | 48. $\sqrt{b} + 1$ .                   |                       | 64. 7.  |                     |                       |
| 35. $\frac{a}{b}$ .                       | 49. $x - y$ .                          |                       | 65. 5.  |                     |                       |
| 36. $\frac{1}{2a}\sqrt{2}$ .              | 50. $ax\sqrt{x} - b\sqrt{y}$ .         |                       | 66. $2\sqrt{3}$ .                                     |                     |                       |
| 37. $3a^2$ .                              | 51. $cy\sqrt[3]{y^2} - d\sqrt[3]{z}$ . |                       | 67. $\frac{1}{2}\sqrt{6}$ .                           |                     |                       |
| 38. $2n\sqrt[3]{2}$ .                     | 52. 3.                                 |                       | 68. $\frac{2}{3}\sqrt{2}$ .                           |                     |                       |
| 39. $7x\sqrt[3]{x^2}$ .                   | 53. $\sqrt[5]{3}$ .                    |                       | 69. $\frac{1}{8}\sqrt{7}$ .                           |                     |                       |
| 40. $2y$ .                                | 54. $\sqrt[5]{7}$ .                    |                       | 70. $\frac{1}{4}\sqrt{210}$ .                         |                     |                       |
| 41. $\frac{1}{x^2}\sqrt[3]{ax^2}$ .       | 55. $\sqrt[5]{24}$ .                   |                       | 71. $9\sqrt{3} + 15$ .                                |                     |                       |
| 42. $5a\sqrt[4]{a}$ .                     | 56. $\sqrt[5]{\frac{1}{4}}$ .          |                       | 72. $6\sqrt{2} + 8$ .                                 |                     |                       |
|   | 57. 0.6.                               |                       | 73. $\frac{1}{a}\sqrt[5]{a^5}$ .                      |                     |                       |

74.  $\sqrt[15]{b}$ . 75.  $\frac{1}{m}\sqrt[12]{m^{11}}$ . 76.  $\sqrt[18]{x^{11}}$ .
77.  $\frac{1}{x}\sqrt[6]{\frac{4}{x}} = \frac{1}{x^{\frac{7}{6}}}\sqrt[6]{4x^5}$ .
78.  $\sqrt[35]{\frac{y^{13}}{32}}$ . 82.  $a\sqrt{a}$ . 87.  $\frac{1}{2x^2y^3}\sqrt[4]{8x^3y^3}$ .
79.  $\frac{1}{6z}\sqrt{2}$ . 83.  $b^2\sqrt{b}$ . 88.  $\frac{2a}{5x}\sqrt[3]{\frac{5a^3}{x}}$ .
80.  $\frac{1}{a}\sqrt[7]{a^{68}b^{30}}$ . 84.  $x^4\sqrt{z}$ . 89.  $3\sqrt{\frac{x+y}{x-y}}$ .
81.  $\sqrt[42]{\frac{a^5}{c^{10}}}$ . 85.  $\frac{y^3}{2}\sqrt{6y}$ . 90.  $\frac{1}{2}\sqrt[4]{m^3} + \frac{2}{3}\sqrt[4]{mn^3}$ .
91.  $\frac{1}{8}$ . 101.  $\frac{1}{a}$ . 108.  $a^4$ .
92. 4. 102.  $\frac{n^3}{m^2\sqrt[3]{m}}$ . 109.  $\sqrt[3]{\frac{1}{m^4}}$ .
93.  $\frac{5}{2}$ . 103.  $\frac{b^2\sqrt{b}}{a\sqrt{a}}$ . 110.  $\sqrt[6]{a^5}$ .
94.  $\frac{1}{\sqrt{6}}$ . 104.  $a$ . 111.  $x^2 \times \sqrt[10]{x^7}$ .
95. 17. 105.  $m^{15}\sqrt{m}$ . 112.  $\frac{1}{\sqrt[15]{a}}$ .
96.  $\frac{1}{8}$ . 106.  $\frac{1}{x^5\sqrt[6]{x}}$ . 113.  $4\sqrt[3]{4}$ .
97. 4. 107.  $\sqrt[10]{\frac{1}{a^{11}}}$ . 114.  $24\sqrt{3}$ .
98.  $\frac{1}{8}$ . 115.  $\sqrt[3]{9}$ .
99.  $\frac{1}{\sqrt{a}}$ .
100.  $\frac{1}{m^2}$ .

Ex. 47. Page 159.

1.  $a^3$ . 3.  $n^3$ . 5.  $y^2$ . 7.  $\sqrt{m^{15}}$ .
2.  $m^2$ . 4.  $x^3$ . 6.  $ab^3$ . 8.  $n^2\sqrt[3]{n^2}$ .



- |   |  |   |
|---|--|---|
| 9. $y\sqrt[4]{y}$ .   | 14. $3a^3$ .                             | 19. $(x-y)\sqrt[5]{(x-y)^3}$            |
| 10. $b^3c\sqrt[3]{b^2c}$ .  | 15. $a^2c$ .                             | 20. $(a^3 - b^3)^2$ .                   |
| 11. $xz^3\sqrt{x^2z^5}$ .   | 16. $2a^4\sqrt[4]{a}$ .                  | 21. $\sqrt[4]{(1+a^2)^3}$ .             |
| 12. $a^3\sqrt[3]{4a^2}$ .   | 17. $a^2x^3$ .                           | 22. $(x^2 - y^2)^{2n}$ .                |
| 13. $1\frac{1}{2}$ .  | 18. $(x+y)^6$ .                          |   |
| 23. $\sqrt[3]{a^4b^3} + m^3 + \sqrt{ax^3} - 2\sqrt[6]{a^4b^3m^3} + 2\sqrt[12]{a^{11}b^4x^3} - 2\sqrt[4]{m^6ax^3}$ . |  |   |
| 24. $\sqrt[2]{a^4} + \sqrt[2]{b^2} - 2\sqrt[2]{a^2b^2}$ .   |  |   |
| 25. $\sqrt[2]{m^2} + \sqrt[2]{n^2} + \sqrt[2]{p^2} - 2\sqrt[2]{m^2n^2} + 2\sqrt[2]{m^2p^2} - 2\sqrt[2]{n^2p^2}$ .   |  |   |
| 26. $2a^3b$ .   | 39. $v^3 = \sqrt[3]{b^3}$ .              | 52. $\sqrt[12]{m^5}$ .                  |
| 27. $-xy^2$ .   | 40. $(x^4)^3 = \sqrt[3]{x^4} = x^{12}$ . | 53. $\sqrt{\sqrt[3]{125}} = \sqrt{5}$ . |
| 28. $(x^2 - y^2)^2$ .   | 41. $\sqrt[4]{x^3}$ .                    | 54. $\sqrt{3}$ .                        |
| 29. $-(a^2 - b^2)^3$ .  | 42. $\sqrt[3]{y^3} = y^3$ .              | 55. $x - y$ .                           |
| 30. 5.  | 43. $\sqrt[3]{a}$ .                      | 56. $(a - b)^2$ .                       |
| 31. 16.   | 44. $\sqrt[6]{b}$ .                      | 57. $(m + n)^2$ .                       |
| 32. 127.  | 45. $\sqrt[12]{c}$ .                     | 58. $\sqrt[3]{(3a - 2b)^2}$ .           |
| 33. -131.   | 46. $\sqrt[30]{2m}$ .                    | 59. $(1 - x^2)^6$ .                     |
| 34. $9\sqrt[3]{9}$ .  | 47. $\sqrt[3]{3a^3}$ .                   | 60. $\sqrt[3]{2x^3}$ .                  |
| 35. $-4\sqrt[4]{2}$ .   | 48. $\sqrt[3]{a}$ .                      | 61. $a^3\sqrt[3]{2}$ .                  |
| 36. $-13^2 = 169$ .   | 49. $\sqrt{m}$ .                         | 62. $m\sqrt[3]{n^2}$ .                  |
| 37. 6.  | 50. $\sqrt[3]{y}$ .                      | 63. $m^{13}\sqrt[3]{6}$ .               |
| 38. $\sqrt{n}$ .  | 51. $\sqrt[3]{a^3}$ .                    | 64. $2a^3\sqrt[3]{3}$ .                 |

## Ex. 48. Page 160.

1.  $\frac{2(3 + \sqrt{6})}{3}$
2.  $\frac{21(4 + \sqrt{6})}{10}$
3.  $4(5\sqrt{3} - 3\sqrt{5})$
4.  $\frac{3(8\sqrt{7} - 7\sqrt{2})}{2}$
5.  $40(3\sqrt{2} + \sqrt{3})$
6.  $4 - \sqrt{15}$
7.  $\frac{8 + \sqrt{55}}{3}$
8.  $\frac{23 - 2\sqrt{130}}{3}$
9.  $\frac{11 + 3\sqrt{14}}{5}$
10.  $4 + \sqrt{15}$
11.  $-\frac{12 + 9\sqrt{2} + 20\sqrt{3} + 15\sqrt{6}}{2}$
12.  $\frac{3 - \sqrt{3} - 12\sqrt{5} + 4\sqrt{15}}{12}$
13.  $\frac{7(4\sqrt{5} - 5) + 15(\sqrt{10} - 4\sqrt{2})}{55}$
14.  $\frac{5(2\sqrt{3} + \sqrt{6}) - 6(1 + \sqrt{2})}{57}$
15.  $-\frac{7(2 + \sqrt{11}) + 3(2\sqrt{7} + \sqrt{77})}{42}$
16.  $\frac{18(120\sqrt{6} + 205\sqrt{3} - 219\sqrt{2} - 308)}{529}$
17.  $\frac{\sqrt{2} + \sqrt{6}}{2}$
18.  $\sqrt{3} + \sqrt{2}$
19.  $2\sqrt{2} + \sqrt{5} - \sqrt{10} - 2$
20.  $\frac{195 - 75\sqrt{3} - 187\sqrt{2} + 137\sqrt{6}}{94}$
21.  $\frac{1}{2}(19\sqrt{2} + 16\sqrt{3} - 10\sqrt{5} - 10\sqrt{6} + 8\sqrt{10} + 8\sqrt{15} - 5\sqrt{30} - 22)$

## Ex. 49. Page 161.

1.  $2\sqrt{-1}$ .
2.  $5\sqrt{-1}$ .
3.  $9\sqrt{-1}$ .
4.  $12\sqrt{-1}$ .
5.  $a\sqrt{-1}$ .
6.  $b^2\sqrt{-1}$ .
7.  $2\sqrt[4]{-1} = \sqrt{2}(1 + \sqrt{-1})$ .
8.  $3\sqrt[4]{-1} = \frac{3}{2}\sqrt{2}(1 + \sqrt{-1})$ .
9.  $5\sqrt[4]{-1} = \frac{5}{2}\sqrt{2}(1 + \sqrt{-1})$ .
10.  $2\sqrt[6]{-1} = 2\sqrt{-1}$ .
11.  $3\sqrt[6]{-1} = 3\sqrt{-1}$ .
12.  $2\sqrt[8]{-1} = \sqrt{2 + \sqrt{2}} + \sqrt{2 - \sqrt{2}}\sqrt{-1}$ .
13.  $x^2\sqrt[4]{-1} = \frac{x^2}{2}\sqrt{2}(1 + \sqrt{-1})$ .
14.  $y^2\sqrt[4]{y}\sqrt[4]{-1} = y^2\sqrt[4]{\frac{y}{4}}(1 + \sqrt{-1})$ .
15.  $z^2\sqrt{-1}$ .
16.  $u^2\sqrt{-1}$ .
17.  $\frac{1}{2}\sqrt{-1}$ .
18.  $\frac{a^2}{b}\sqrt{-1}$ .
19.  $\sqrt{x^2 + y^2}\sqrt{-1}$ .
20.  $\sqrt{x^2 + y^2}\sqrt[4]{-1} = \sqrt{\frac{x^2 + y^2}{2}}(1 + \sqrt{-1})$ .
21.  $(3x - 2y^2)^2\sqrt{-1}$ .
22.  $3x^3\sqrt{-1}$ .
23.  $9m^3\sqrt{-1}$ .
24.  $-1; 1; 1; 1; \sqrt{-1}; -\sqrt{-1}; -\sqrt{-1}$ .
25.  $-\sqrt{-1}; -\sqrt{-1}; 1; 1; -1; \sqrt{-1}; -1$ .
26.  $10\sqrt{-1}$ .
27.  $(3a^2 - 7a)\sqrt{-1}$ .
28.  $2(x - y + 4xy)\sqrt{-1}$ .
29.  $(10a^3 - a^4)\sqrt{-1}$ .

- 
- |   |                                  |                    |
|---|----------------------------------|--------------------|
| 30. $m(1 + 2n)i$ .                                    | 41. $3m^3$ .                     |                    |
| 31. $11 - 3i$ .                                       | 42. $-z$ .                       |                    |
| 32. $\frac{25a - 33b}{2}\sqrt{-1}$ .                  | 43. $-6$ .                       |                    |
| 33. $2[13 + (1 + b)i]$ .                              | 44. $8\sqrt{14}$ .               |                    |
| 34. $-a$ .  | 45. $77$ .                       |                    |
| 35. $-12$ .   | 46. $-\sqrt{a^2 - b^2}$ .        |                    |
| 36. $-\sqrt{mn}$ .                                    | 47. $(x - y)\sqrt{-1}$ .         |                    |
| 37. $-\sqrt{y}$ .                                     | 48. $(y - 5)\sqrt{-1}$ .         |                    |
| 38. $35\sqrt{-1}$ .                                   | 49. $-3\sqrt{2 - y}$ .           |                    |
| 39. $a^2\sqrt{-1}$ .                                  | 50. $a^2 + 1$ .                  |                    |
| 40. $-12yz^3$ .                                       | 51. $m^2 - b^2i^2 = m^2 + b^2$ . |                    |
|   | 52. $47 - \sqrt{-1}$ .           |                    |
| 53. $xy + 2b + (2y - bx)\sqrt{-1}$ .                  |                                  |                    |
| 54. $6(xy + 4) + (9x - 16y)\sqrt{-1}$ .               |                                  |                    |
| 55. $2ac + 3bd + (3bc - 2ad)\sqrt{-1}$ .              |                                  |                    |
| 56. $16 - 2i\sqrt{2}$ .                               |                                  |                    |
| 57. $mn + 12\sqrt{bc} + (4m\sqrt{c} - 3n\sqrt{b})i$ . |                                  |                    |
| 58. $2(\sqrt{15} + 2\sqrt{3}) + 2(3 - \sqrt{10})i$ .  |                                  |                    |
| 59. $-a\sqrt{-1}$ .                                   | 63. $\sqrt{\frac{m}{n}}$ .       | 67. $-i\sqrt{m}$ . |
| 60. $-\sqrt{-1}$ .                                    | 64. $\sqrt{a}$ .                 | 68. $\sqrt{x}$ .   |
| 61. $-\frac{di}{2}$ .                                 | 65. $-i\sqrt{x}$ .               | 69. $2$ .          |
| 62. $\sqrt{3}$ .                                      | 66. $-\sqrt{-1}$ .               | 70. $m\sqrt{2}$ .  |

## Ex. 50. Page 163.

1. 2.	26. 2.	51. 4.	75. 8; $4\frac{4}{17}$ .
2. 3.	27. $-\frac{5}{7}$ .	52. 4.	76. 4; $-\frac{4}{7}$ .
3. $2\frac{5}{8}$ .	28. $-3$ .	53. $\frac{a}{a+2\sqrt{a}}$ .	77. $\frac{1}{2}$ ; $-4\frac{1}{8}$ .
4. $12\frac{1}{4}$ .	29. 1.	54. $\sqrt{3}$ ; 0.	78. $\frac{5}{4}$ ; $55\frac{1}{4}$ .
5. $\frac{1}{8}$ .	30. $\frac{1}{4}$ .	55. $\frac{1}{2}\sqrt{3}$ .	79. 4; $-\frac{8}{9}$ .
6. $\frac{1}{4}$ .	31. 49.	56. 3.	80. $-4$ ; $-7$ .
7. $\frac{4}{5}$ .	32. 5.	57. 6; 2.	81. 3; $\frac{9}{10}$ .
8. $-\frac{1}{8}$ .	33. 23.	58. 7; 3.	82. 2; $-16\frac{1}{2}$ .
9. $\frac{1}{20}$ .	34. 8.	59. 4; $-25$ .	83. 1; $-\frac{11}{24}$ .
10. 2.	35. 6.	60. 196; 49.	84. 3; $-3\frac{5}{7}$ .
11. $-2$ .	36. 5.	61. $1\frac{8}{15}$ ; $\frac{2}{3}$ .	85. 7; $-6$ .
12. $-\frac{1}{8}$ .	37. 49.	62. 2; $\frac{1}{2}$ .	86. $\frac{1}{2}$ ; $-\frac{5}{7}$ .
13. $6\frac{2}{3}$ .	38. 4.	63. 5; $-2$ .	87. 1; $-\frac{1}{8}$ .
14. $-2\frac{1}{2}$ .	39. 4.	64. $6\frac{9}{10}$ ; 6.	88. 5; $-4\frac{11}{18}$ .
15. $-6$ .	40. $8\frac{1}{2}$ .	65. 7; 0.	89. 3; $-\frac{2}{3}$ .
16. 2.	41. 60.	66. 2; $-\frac{5}{9}$ .	90. $\pm 2$ .
17. 5.	42. $4\frac{1}{2}$ .	67. 6; $1\frac{4}{5}$ .	91. $\pm \frac{1}{2}$ .
18. $-5$ .	43. $\frac{1}{9}$ .	68. 5; $\frac{5}{8}$ .	92. $\frac{2}{3}$ ; $\frac{1}{5}$ .
19. $1\frac{3}{5}$ .	44. $-\frac{1}{4}$ .	69. 2; $-\frac{1}{10}$ .	93. 4; $\frac{1}{2}$ .
20. $-5$ .	45. $\frac{1}{8}$ .	70. 9; 1.	94. $\pm 2$ .
21. 1.	46. 25.	71. 4; $\frac{8}{9}$ .	95. $\pm \frac{2}{3}$ .
22. 4.	47. $\frac{3}{4}$ .	72. 3; $-\frac{11}{8}$ .	96. 4; $-\frac{1}{4}$ .
23. $\frac{7}{9}$ .	48. $\frac{1}{8}$ .	73. 5; 1.	97. $\frac{b}{a}$ ; $-1$ .
24. $2\frac{1}{2}$ .	49. $\frac{1}{8}$ .	74. $\frac{1}{2}$ ; $-\frac{9}{10}$ .	98. $\frac{4a}{3}$ ; $\frac{a}{3}$ .
25. $\frac{1}{2}$ .	50. $200\frac{1}{2}$ .		

99.  $\frac{1}{2a}(1 \pm \sqrt{1+4b^2})$ .      102.  $2a; -\frac{2a}{3}$   
 100.  $\frac{1}{2}(\sqrt{a} \pm \sqrt{b})$ .      103.  $4a; -2a$ .  
 101.  $\frac{a}{4}(1 \pm \sqrt{7})$ .      104.  $\frac{a}{3}; -\frac{a+2b}{3}$ .  
 105.  $\frac{a(2 \pm \sqrt{2})}{2}$ .      110.  $\pm \frac{a}{2}\sqrt{\frac{a^2-4}{a^2-1}}$ .      113.  $\frac{a}{2b^2}; -\frac{9a}{2b^2}$ .  
 106.  $\pm \frac{2a}{\sqrt{5}}$ .      111.  $0; \frac{9a}{16}$ .      114.  $0; a$ .  
 107.  $b \pm \sqrt{b^2-ab}$ .      115.  $\pm \sqrt{ab}$ .  
 108.  $\pm \sqrt{n}$ .      112.  $\pm a\sqrt{\frac{a}{a-1}}$ .      116.  $\frac{a}{1+\sqrt[3]{a^2}}$ .  
 109.  $\pm 2b$ .  
 117.  $\frac{a+b}{2} \pm \frac{a-b}{4}\sqrt{2}$ .  
 118.  $\pm 2a$  are evidently the two roots.  
 119. The roots are evidently  $a$  and  $b$ .  
 120.  $\frac{a-b}{2} \pm \frac{(a+b)c}{2\sqrt{c^2+4}}$ .      122.  $\frac{a+b}{2} \pm \frac{a-b}{2c}\sqrt{c^2+4}$ .  
 121.  $\frac{a+b}{2} \pm \frac{(a-b)c}{2\sqrt{c^2-4}}$ .      123.  $x=6a$ , or  $-\frac{2a}{3}$ .

## Ex. 51. Page 169.

- |                 |                 |                  |
|-----------------|-----------------|------------------|
| 1. \$3900.48.   | 5. \$1377.01.   | 9. \$124,225.88. |
| 2. \$13,823.13. | 6. \$8,646,290. | 10. \$5,643.20.  |
| 3. \$4855.87.   | 7. \$6250.      | 11. \$3,129.29.  |
| 4. \$35,705.48. | 8. \$368,429.   | 12. \$606.77.    |
| 13. \$1459.52.  |                 |                  |

$$47. \frac{a^6}{64} + \frac{3a^5z}{16} + \frac{15a^4z^2}{16} + \frac{5a^3z^3}{2} + \frac{15a^2z^4}{4} + 3az^5 + z^6.$$

$$48. \frac{b^8}{6561} - \frac{8b^7y}{2187} + \frac{28b^6y^2}{729} - \frac{56b^5y^3}{243} + \frac{70b^4y^4}{81} - \frac{56b^3y^5}{27} \\ + \frac{28b^2y^6}{9} - \frac{8by^7}{3} + y^8.$$

$$49. \frac{512x^2}{19683} + \frac{256x^2y}{729} + \frac{512x^2y^2}{243} + \frac{1792x^2y^3}{243} + \frac{448x^2y^4}{27} \\ + \frac{224x^2y^5}{9} + \frac{224x^2y^6}{9} + 16x^2y^7 + 6xy^8 + y^9.$$

$$50. \frac{177147a^{11}}{4194304} - \frac{649539a^{10}b}{1048576} + \frac{1082565a^9b^2}{262144} - \frac{1082565a^8b^3}{65536} \\ + \frac{360855a^7b^4}{8192} - \frac{168399a^6b^5}{2048} + \frac{56133a^5b^6}{512} \\ - \frac{13365a^4b^7}{128} + \frac{4455a^3b^8}{64} - \frac{495a^2b^9}{16} + \frac{33ab^{10}}{4} - b^{11}.$$

$$51. x^4 - 2.4x^3 + 2.16x^2 - 0.864x + 0.1296.$$

$$52. -x^7 + 14x^6a - 84x^5a^2 + 280x^4a^3 - 560x^3a^4 + 672x^2a^5 \\ - 448xa^6 + 128a^7.$$

$$53. 6561x^8 - 17,496x^7b + 20,412x^6b^2 - 13,608x^5b^3 \\ + 5670x^4b^4 - 1512x^3b^5 + 252x^2b^6 - 24xb^7 + b^8.$$

$$54. -y^9 + 18y^8c - 144y^7c^2 + 672y^6c^3 - 2016y^5c^4 + 4032y^4c^5 \\ - 5376y^3c^6 + 4608y^2c^7 - 2304yc^8 + 512c^9.$$

$$55. -z^5 + 15z^4a - 90z^3a^2 + 270z^2a^3 - 405za^4 + 243a^5.$$

$$56. 64x^6 + 576x^5a + 2160x^4a^2 + 4320x^3a^3 + 4860x^2a^4 \\ + 2916xa^5 + 729a^6.$$

$$57. 3125d^5 - 9375d^4y + 11,250d^3y^2 - 6750d^2y^3 + 2025dy^4 \\ - 243y^5.$$

58.  $16,384d^7 - 86,016d^6z + 193,536d^5z^2 - 241,920d^4z^3$   
 $+ 181,440d^3z^4 - 81,648d^2z^5 + 20,412dz^6 - 2187z^7.$
59.  $0.00032a^5 + 0.0024a^4b + 0.0072a^3b^2 + 0.0108a^2b^3$   
 $+ 0.0081ab^4 + 0.00243b^5.$
60.  $3.5831808b^7 - 4.1803776b^6y + 2.0901888b^5y^2$   
 $- 0.580608b^4y^3 + 0.096768b^3y^4 - 0.0096768b^2y^5$   
 $+ 0.0005376by^6 - 0.0000128y^7.$
61.  $\frac{a^5}{243} + \frac{5a^4b}{162} + \frac{10a^3b^2}{108} + \frac{10a^2b^3}{72} + \frac{5ab^4}{48} + \frac{b^5}{32}.$
62.  $\frac{c^8}{390625} - \frac{2c^7d}{78125} + \frac{7c^6d^2}{62500} - \frac{7c^5d^3}{25000} + \frac{7c^4d^4}{16000} - \frac{7c^3d^5}{16000}$   
 $+ \frac{7c^2d^6}{25600} - \frac{2cd^7}{20480} + \frac{d^8}{65536}.$
63.  $\frac{512m^9}{1953125} + \frac{1728m^8p}{390625} + \frac{2592m^7p^2}{78125} + \frac{2268m^6p^3}{15625}$   
 $+ \frac{5103m^5p^4}{12500} + \frac{15309m^4p^5}{20000} + \frac{15309m^3p^6}{16000} + \frac{19683m^2p^7}{25600}$   
 $+ \frac{59049mp^8}{163840} + \frac{19683p^9}{262144}.$
64.  $\frac{64z^6}{15625} - \frac{576z^5b}{21875} + \frac{2160z^4b^2}{30625} - \frac{864z^3b^3}{8575} + \frac{972z^2b^4}{12005}$   
 $- \frac{2916zb^5}{84035} + \frac{729b^6}{117649}.$
65.  $\frac{2187h^7}{78125} - \frac{5103h^6v}{31250} + \frac{5103h^5v^2}{12500} - \frac{2835h^4v^3}{5000} + \frac{945h^3v^4}{2000}$   
 $- \frac{189h^2v^5}{800} + \frac{21hv^6}{320} - \frac{v^7}{128}.$
66.  $a^{10} + 5a^9b^3 + 10a^8b^6 + 10a^7b^9 + 5a^6b^{12} + b^{15}.$
67.  $x^{18} - 6x^{15}a^3 + 15x^{12}a^{10} - 20x^9a^{15} + 15x^6a^{20} - 6x^3a^{25} + a^{30}.$



68.  $y^{14} + 7y^{12}b + 21y^{10}b^2 + 35y^8b^3 + 35y^6b^4 + 21y^4b^5 + 7y^2b^6 + b^7.$
69.  $d^{24} - 8d^{21}b^2 + 28d^{18}b^4 - 56d^{15}b^6 + 70d^{12}b^8 - 56d^9b^{10} + 28d^6b^{12} - 8d^3b^{14} + b^{16}.$
70.  $a^{27} - 9a^{24}y^2 + 36a^{21}y^4 - 84a^{18}y^6 + 126a^{15}y^8 - 126a^{12}y^{10} + 84a^9y^{12} - 36a^6y^{14} + 9a^3y^{16} - y^{18}.$
71.  $d^{24} - 24d^{21}x^2 + 252d^{18}x^4 - 1512d^{15}x^6 + 5670d^{12}x^8 - 13,608d^9x^{10} + 20,412d^6x^{12} - 17,496d^3x^{14} + 6561x^{16}.$
72.  $32h^{10} - 240h^8x^2 + 720h^6x^4 - 1080h^4x^6 + 810h^2x^{12} - 243x^{15}.$
73.  $2187a^{14} + 25,515a^{12}b^2 + 127,575a^{10}b^4 + 354,375a^8b^6 + 590,625a^6b^{12} + 590,625a^4b^{15} + 328,125a^2b^{18} + 78,125b^{21}.$
74.  $1 - 10x^2 + 40x^4 - 80x^6 + 80x^{12} - 32x^{15}.$
75.  $6561x^{16} - 17,496x^{14} + 20,412x^{12} - 13,608x^{10} + 5670x^8 - 1512x^6 + 252x^4 - 24x^2 + 1.$
76.  $1 + \frac{16a^5}{3} + \frac{112a^{10}}{9} + \frac{448a^{15}}{27} + \frac{1120a^{20}}{81} + \frac{1792a^{25}}{243} + \frac{1792a^{30}}{729} + \frac{1024a^{35}}{2187} + \frac{256a^{40}}{6561}.$
77.  $\frac{p^{25}}{32} + \frac{15p^{20}y^4}{16} + \frac{90p^{15}y^8}{8} + \frac{270p^{10}y^{12}}{4} + \frac{405p^5y^{16}}{2} + 243y^{20}.$
78.  $\frac{32a^{15}}{243} + \frac{20a^{12}b^2}{27} + \frac{5a^9b^4}{3} + \frac{15a^6b^6}{8} + \frac{135a^3b^8}{128} + \frac{243b^{10}}{1024}.$
79.  $0.0016a^{12} - \frac{0.064a^9x^4}{3} + \frac{0.32a^6x^8}{3} - \frac{6.4a^3x^{12}}{27} + \frac{16x^{16}}{81}.$
80.  $\frac{78125l^{14}}{2187} + \frac{10937.5l^{12}p^2}{243} + \frac{218.75l^{10}p^6}{9} + \frac{21.875l^8p^9}{3} + 1.3125l^6p^{12} + 0.14175l^4p^{15} + 0.008505l^2p^{18} + 0.0002187p^{21}.$

81.  $128a^7 - 1344a^6cx + 6048a^5c^2x^2 - 15,120a^4c^3x^3$   
 $+ 22,680a^3c^4x^4 - 20,412a^2c^5x^5 + 10,206ac^6x^6$   
 $- 2187c^7x^7.$
82.  $729a^{12} + 2916a^{10}c^2x^2 + 4860a^8c^4x^4 + 4320a^6c^6x^6$   
 $+ 2160a^4c^8x^8 + 576a^2c^{10}x^{10} + 64c^{12}x^{12}.$
83.  $b^5c^{10} - 10ab^4c^8y^2 + 40a^2b^3c^6y^4 - 80a^3b^2c^4y^6 + 80a^4bc^2y^{12}$   
 $- 32a^5y^{15}.$
84.  $256c^{16} - 768ac^{13}x^5 + 864a^2c^{10}x^{10} - 432a^3c^7x^{15} + 81a^4c^4x^{20}.$
85.  $a^{10}b^{15} + 10a^{11}b^{13}x^4 + 40a^{12}b^{11}x^8 + 80a^{13}b^9x^{12} + 80a^{14}b^7x^{16}$   
 $+ 32a^{15}b^5x^{20}.$
86.  $\frac{128a^7b^{14}}{2187} - \frac{112a^8b^{12}y}{243} + \frac{14a^9b^{10}y^2}{9} - \frac{35a^{10}b^8y^3}{12}$   
 $+ \frac{105a^{11}b^6y^4}{32} - \frac{567a^{12}b^4y^5}{256} + \frac{1701a^{13}b^2y^6}{2048} - \frac{2187a^{14}y^7}{16384}.$
87.  $a^3\sqrt{a} + 7a^3x + 21a^2\sqrt{a} \times x^2 + 35a^2x^3 + 35a\sqrt{a}$   
 $\times x^4 + 21ax^5 + 7\sqrt{a} \times x^6 + x^7.$
88.  $16b^4 - 64b^3\sqrt{2b} \times m + 224b^3m^2 - 224b^2\sqrt{2b}$   
 $\times m^3 + 280b^2m^4 - 112\sqrt{2b} \times m^5 + 56bm^6 - 8\sqrt{2b}$   
 $\times m^7 + m^8.$
89.  $27c^3\sqrt{3c} + 378c^3a + 756c^2\sqrt{3c} \times a^2 + 2520c^2a^3$   
 $+ 1680c\sqrt{3c} \times a^4 + 2016ca^5 + 448\sqrt{3c} \times a^6 + 128a^7.$
90.  $\frac{a^4}{16} - 3a^3\sqrt{\frac{1}{2}a} \times y + \frac{63a^3y^2}{2} - 378a^2\sqrt{\frac{1}{2}a} \times y^3$   
 $+ \frac{2835a^2y^4}{2} - 6804a\sqrt{\frac{1}{2}a} \times y^5 + 10,206ay^6$   
 $- 17,496\sqrt{\frac{1}{2}a} \times y^7 + 6561y^8.$
91.  $64e^3 + 192ae^2\sqrt{e} + 240a^2e^2 + 160a^3e\sqrt{e} + 60a^4e$   
 $+ 12a^5\sqrt{e} + a^6.$

$$92. 512b^3 - 2304b^2\sqrt{x} + 4608b^1x - 5376b^0x\sqrt{x} \\ + 4032b^5x^2 - 2016b^4x^2\sqrt{x} + 672b^3x^3 - 144b^2x^3\sqrt{x} \\ + 18bx^4 - x^4\sqrt{x}.$$

$$93. \frac{64a^6}{729} + \frac{64a^5\sqrt{2x}}{81} + \frac{160a^4x}{27} + \frac{320a^3x\sqrt{2x}}{27} + \frac{80a^2x^2}{3} \\ + 16ax^3\sqrt{2x} + 8x^3.$$

$$94. \frac{243a^5}{1024} - \frac{405a^4\sqrt{\frac{1}{2}x}}{256} + \frac{135a^3x}{64} - \frac{45a^2x\sqrt{\frac{1}{2}x}}{16} \\ + \frac{15ax^2}{16} - \frac{x^2\sqrt{\frac{1}{2}x}}{4}.$$

$$95. 64a^6 - 576a^5\sqrt{y} + 2160a^4y - 4320a^3y\sqrt{y} + 4860a^2y^2 \\ - 2916ay^2\sqrt{y} + 729y^3.$$

$$96. a^{14} + \frac{7a^{12}\sqrt{z}}{2} + \frac{21a^{10}z}{4} + \frac{35a^8z\sqrt{z}}{8} + \frac{35a^6z^2}{16} \\ + \frac{21a^4z^2\sqrt{z}}{32} + \frac{7a^2z^3}{64} + \frac{z^3\sqrt{z}}{128}.$$

$$97. a^4\sqrt{a} + 9a^4\sqrt{x} + 36a^3x\sqrt{a} + 84a^3x\sqrt{x} \\ + 126a^2x^2\sqrt{a} + 126a^2x^2\sqrt{x} + 84ax^3\sqrt{a} \\ + 36ax^3\sqrt{x} + 9x^4\sqrt{a} + x^4\sqrt{x}.$$

$$98. b^4 - 8b^3\sqrt{by} + 28b^2y - 56b^2y\sqrt{by} + 70b^2y^2 \\ - 56by^2\sqrt{by} + 28by^3 - 8y^3\sqrt{by} + y^4.$$

$$99. 8c^3 + 24c^2\sqrt{6cx} + 180c^2x + 120cx\sqrt{6cx} + 270cx^2 \\ + 54x^2\sqrt{6cx} + 27x^3.$$

$$100. \frac{a^3\sqrt{\frac{1}{2}a}}{8} - \frac{7a^3\sqrt{2x}}{8} + \frac{21a^2x\sqrt{\frac{1}{2}a}}{2} - \frac{35a^2x\sqrt{2x}}{2} \\ + 70ax^2\sqrt{\frac{1}{2}a} - 42ax^2\sqrt{2x} + 56x^3\sqrt{\frac{1}{2}a} - 8x^3\sqrt{2x}.$$

101.  $\frac{8a^3}{27} + \frac{8a^2\sqrt{\frac{1}{3}ax}}{3} + 5a^2x + 10ax\sqrt{\frac{1}{3}ax} + \frac{45ax^2}{8}$   
 $+ \frac{27x^2\sqrt{\frac{1}{3}ax}}{8} + \frac{27x^3}{64}.$
102.  $a^{15} - 10a^{13}x^2\sqrt{ax} + 45a^{11}x^5 - 120a^{10}x^7\sqrt{ax}$   
 $+ 210a^9x^{10} - 252a^7x^{12}\sqrt{ax} + 210a^6x^{15} - 120a^4x^{17}\sqrt{ax}$   
 $+ 45a^3x^{20} - 10ax^{22}\sqrt{ax} + x^{25}.$
103.  $b^{22}\sqrt{b} + 9b^{20}y^2 + 36b^{17}y^4\sqrt{b} + 84b^{15}y^6 + 126b^{12}y^8\sqrt{b}$   
 $+ 126b^{10}y^{10} + 84b^7y^{12}\sqrt{b} + 36b^5y^{14} + 9b^2y^{16}\sqrt{b} + y^{18}.$
104.  $\frac{8h^9}{27} + \frac{8h^7x^3\sqrt{\frac{1}{3}h}}{3} + \frac{10h^6x^4}{3} + \frac{20h^4x^6\sqrt{\frac{1}{3}h}}{3} + \frac{5h^3x^8}{2}$   
 $+ \frac{3hx^{10}\sqrt{\frac{1}{3}h}}{2} + \frac{x^{12}}{8}.$
105.  $4a^2\sqrt[3]{4a^3} + 32a^2\sqrt[3]{6ax^3} + 112a^2x\sqrt[3]{9x} + 336ax^2\sqrt[3]{4a^3}$   
 $+ 420ax^2\sqrt[3]{6ax^3} + 336ax^2\sqrt[3]{9x} + 252x^4\sqrt[3]{4a^3}$   
 $+ 72x^4\sqrt[3]{6ax^3} + 9x^5\sqrt[3]{9x}.$
106.  $32a^5x^5 + 80a^4x^4\sqrt[3]{3xy^2} + 80a^3x^3y\sqrt[3]{9x^2y} + 120a^2x^2y^2$   
 $+ 30ax^2y^2\sqrt[3]{3xy^2} + 3xy^3\sqrt[3]{9x^2y}.$
107.  $27a^3y^3\sqrt{3ay} + 189a^3y^3\sqrt[3]{2x^2y} + 189a^2xy^2\sqrt[5]{432a^3x^3y}$   
 $+ 630a^2x^2y^3 + 210ax^2y^2\sqrt[5]{108a^3x^4y^5} + 126ax^2y^2\sqrt[3]{4xy^2}$   
 $+ 28x^4y^2\sqrt{3ay} + 4x^4y^2\sqrt[3]{2x^2y}.$
108.  $6561a^3b^3x^{24} - 17,496a^2b^4x^{21}\sqrt[3]{5b^2xy^2}$   
 $+ 20,412a^2b^4x^{18}y\sqrt[3]{25b^2xy} - 68,040a^2b^4x^{16}y^2$   
 $+ 28,350a^2b^6x^{13}y^2\sqrt[3]{5b^2xy^2} - 7560a^3b^6x^{10}y^3\sqrt[3]{25b^2xy}$   
 $+ 6300a^2b^6x^8y^4 - 600ab^5x^5y^4\sqrt[3]{5b^2xy^2}$   
 $+ 25b^5x^2y^5\sqrt[3]{25b^2xy}.$

109.  $\frac{x^5}{a^{10}} + \frac{10x^4\sqrt[6]{x^3y^3}}{a^9b} + \frac{45x^4\sqrt[3]{y^3}}{a^8b^2} + \frac{120x^3y\sqrt{x}}{a^7b^3}$   
 $+ \frac{210x^3y\sqrt[3]{y}}{a^6b^4} + \frac{252x^2y\sqrt[6]{x^3y^4}}{a^5b^5} + \frac{210x^2y^3}{a^4b^6}$   
 $+ \frac{120xy^3\sqrt[6]{x^3y^3}}{a^3b^7} + \frac{45xy^3\sqrt[3]{y^3}}{a^2b^8} + \frac{10y^3\sqrt{x}}{ab^9} + \frac{y^3\sqrt[3]{y}}{b^{10}}.$
110.  $\frac{x^4\sqrt[3]{x^3}}{a^{14}} + \frac{7x^4\sqrt[4]{xy^3}}{a^{12}b} + \frac{21x^3y\sqrt[6]{x^3y^3}}{a^{10}b^2} + \frac{35x^2y^2\sqrt[12]{x^5y^3}}{a^8b^3}$   
 $+ \frac{35x^2y^3}{a^6b^4} + \frac{21x^2y^3\sqrt[12]{x^7y^3}}{a^4b^5} + \frac{7x^2y^4\sqrt[4]{xy^3}}{a^2b^6} + \frac{xy^6\sqrt[4]{x^2y}}{b^7}.$
111.  $\frac{x^3z^3}{y^5} - \frac{6z^3\sqrt[6]{a^3x^3y^4z^3}}{y^6} + \frac{15z^3\sqrt[3]{a^3y}}{x^2y^3} - \frac{20az\sqrt{xz}}{x^5y}$   
 $+ \frac{15az\sqrt[3]{ay^3}}{x^7} - \frac{6ay^3\sqrt[6]{a^4x^3y^3z^3}}{x^{10}} + \frac{a^3y^4}{x^{12}}.$
112.  $128a^7 - 448a^6x\sqrt{-1} - 672a^5x^2 + 560a^4x^3\sqrt{-1}$   
 $+ 280a^3x^4 - 84a^2x^5\sqrt{-1} - 14ax^6 + x^7\sqrt{-1}.$
113.  $729b^{12} + 2916b^{10}x\sqrt{-1} - 4860b^8x^2 - 4320b^6x^3\sqrt{-1}$   
 $+ 2160b^4x^4 + 576b^2x^5\sqrt{-1} - 64x^6.$
114.  $a^7\sqrt{a} + 5a^6i\sqrt{x} - 10a^4x\sqrt{a} - 10a^3xi\sqrt{x}$   
 $+ 5ax^3\sqrt{a} + x^2i\sqrt{x}.$
115.  $a^4 + 6a^3i\sqrt[12]{a^4x^3} - 15a^2x\sqrt[6]{a^4x^3} - 20a^2xi\sqrt[4]{x}$   
 $+ 15ax^3\sqrt[3]{a} + 6xi\sqrt[12]{a^8x^3} - x^4\sqrt{x}.$
116.  $a^6 - 9a^5i\sqrt[3]{a} - 36a^4\sqrt[3]{a^2} + 84a^4i + 126a^3\sqrt[3]{a}$   
 $- 126a^2i\sqrt[3]{a^2} - 84a^2 + 36ai\sqrt[3]{a} + 9\sqrt[3]{a^2} - i.$
117.  $-b^5 - 10b^4\sqrt{b} - 45b^4 - 120b^3\sqrt{b} - 210b^3 - 252b^2\sqrt{b}$   
 $- 210b^2 - 120b\sqrt{b} - 45b - 10\sqrt{b} - 1.$

$$118. (x^6 - 9x^5\sqrt[6]{8x^3} + 72x^4\sqrt[3]{x^3} - 168x^4\sqrt{2} + 504x^3\sqrt[3]{x} \\ - 504x^2\sqrt[6]{8x^3} + 672x^2 - 288x\sqrt[6]{8x^3} + 144\sqrt[3]{x^3} \\ - 16\sqrt{2})i.$$

$$119. -(y^5\sqrt[5]{y^3} + 7y^4\sqrt[15]{5^5y^{12}} + 21y^4\sqrt[3]{25} + 175y^3\sqrt[5]{y} \\ + 175y^2\sqrt[15]{5^5y^3} + 105y\sqrt[15]{25^5y^3} + 175y\sqrt[5]{y^4} + 25\sqrt[3]{5})i.$$

$$120. \frac{256b^{12}}{y^8} - \frac{1024b^8d\sqrt[6]{b^3y^4}}{y^7} + \frac{1792b^7d^2\sqrt[3]{y}}{y^5} - \frac{1792b^4d^3\sqrt{b}}{y^3} \\ + \frac{1120b^2d^4\sqrt[3]{y^3}}{y^2} - \frac{448d^5\sqrt[6]{b^3y^3}}{b} + \frac{112d^6y^3}{b^3} \\ - \frac{16d^7y^3\sqrt[6]{b^3y^4}}{b^6} + \frac{d^8y^5\sqrt[3]{y}}{b^8}.$$

$$121. 8,200,192a^6.$$

$$126. -12,033,222,880y^{21}.$$

$$122. 352,716a^{11}d^{10}.$$

$$127. \frac{1547a^{11}x^9}{256}.$$

$$123. 326,592x^{10}.$$

$$124. -29,753,610,120a^{18}x^{55}.$$

$$128. -165a^4x^3.$$

$$125. 119,759,850x^{24}.$$

$$129. -1287a^4x^5i.$$

$$130. 36,146,147,370,366,245,273,600b^{21}c^{58}.$$

131. The coefficient of the  $n$ th term from the beginning is

$$\frac{m(m-1)(m-2) \dots (m-n+2)}{1 \times 2 \times 3 \dots (n-1)}.$$

If the numerator and denominator are multiplied by  $1 \times 2 \times 3 \dots (m-n+1)$ , this expression becomes

$$\frac{m}{(n-1)(m-n+1)}. \quad (A)$$

The  $n$ th term from the end is the  $(m+1) - (n-1)$ , or the  $m-n+2$  from the beginning, and its coefficient is

$$\frac{m(m-1)(m-2) \dots [m-(m-n+2)+2]}{1 \times 2 \times 3 \dots m-n+1}$$

$$= \frac{m(m-1)(m-2) \dots n}{1 \times 2 \times 3 \dots (m-n+1)}$$

If these two terms are multiplied by  $1 \times 2 \times 3 \dots (n-1)$ , the second member of the last equality becomes

$$\frac{\overline{m}}{\overline{n-1} \overline{m-n+1}},$$

an expression equal to  $(A)$ .

132. The development of  $(a+b)^m$  includes  $m+1$  terms, of which the  $n$ th is

$$\frac{m(m-1)(m-2) \dots (m-n+2)}{1 \times 2 \times 3 \dots (n-1)} a^{m-n+1} b^{n-1}.$$

The coefficient of the  $(n+1)$  term will be formed by multiplying the preceding coefficient by  $\frac{m-n+1}{n}$ .

This product will increase so much that we shall have  $\frac{m-n+1}{n} > 1$  or  $n < \frac{m+1}{2}$ . The maximum will

have  $\frac{m+1}{2}$  for  $n$ . As  $n$  is integral, if  $m$  is even,

the term affected with the greatest coefficient will be marked by the integral number immediately superior to  $\frac{m+1}{2}$ . If  $m$  is uneven, there

will be two terms whose coefficients will be maximum, namely, the  $\left(\frac{m+1}{2}\right)$  term from the beginning and following it, the  $\left(\frac{m+1}{2}\right)$  term from the end.

133. The greatest coefficient is :

1. Of  $(a + b^3)^{21}$ , the eleventh and twelfth = 352,716.
2. Of  $(b + x^2)^{16}$ , the ninth = 12,870.
3. Of  $(1 + y^5)^{32}$ , the seventeenth = 601,080,390.

134. The coefficients of the development of  $(1 + a)^m$  are identically the same as those of the development of  $(1 - a)^m$ . Then we have

$$(1 - a)^m = 1 - \frac{m}{1}a + \frac{m(m-1)}{1 \times 2}a^2 - \frac{m(m-1)(m-2)}{1 \times 2 \times 3}a^3 + \dots$$

When  $m$  is entire and positive, the terms of uneven rank are all positive, and those of even rank negative. If  $a = 1$ , the two members cancel each other. Then the sum of the terms of even rank equals that of the terms of uneven rank.

$$135. (1 + x)^m = 1 + \frac{m}{1}x + \frac{m(m-1)}{1 \times 2}x^2 + \frac{m(m-1)(m-2)}{1 \times 2 \times 3}x^3 + \dots$$

$$\text{or } (1 + x)^m - 1 = \frac{m}{1}x + \frac{m(m-1)}{1 \times 2}x^2 + \frac{m(m-1)(m-2)}{1 \times 2 \times 3}x^3 + \dots$$

Making  $x = 1$ , we have

$$2^m - 1 = \frac{m}{1} + \frac{m(m-1)}{1 \times 2} + \frac{m(m-1)(m-2)}{1 \times 2 \times 3} + \dots$$

$$136. (1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$$

$$(i.) 1 - \frac{1}{9} + \frac{1}{9^2} - \frac{1}{9^3} + \frac{1}{9^4} - \frac{1}{9^5} + \dots = 0.9.$$



$$(ii.) 1 - \frac{7}{100} + \left(\frac{7}{100}\right)^2 - \left(\frac{7}{100}\right)^3 + \left(\frac{7}{100}\right)^4 - \left(\frac{7}{100}\right)^5 + \dots$$

$$= \frac{3100}{3317} = 0.93457943\dots$$

$$(iii.) 1 + \frac{9}{1000} + \left(\frac{9}{1000}\right)^2 + \left(\frac{9}{1000}\right)^3 + \dots = \frac{1000}{991}$$

$$= 1.009081735\dots$$

$$137. (1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \frac{7x^5}{256} - \dots$$

$$= 1 + \frac{3}{16} - \frac{9}{512} + \frac{27}{8192} - \frac{320}{524108} + \frac{1701}{8388608} - \dots$$

$$= 1.1726.$$

$$138. (1+x)^{-\frac{1}{2}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \frac{63x^5}{256} + \dots$$

$$= 0.998503367\dots$$

$$139. (1+x)^{\frac{1}{3}} = 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} - \frac{10x^4}{243} + \frac{22x^5}{729} - \frac{154x^6}{6561} + \dots$$

(i.) 1.16553; (ii.) 0.96905; (iii.) 1.04004.

$$140. (1+x)^{-\frac{1}{3}} = 1 - \frac{x}{3} + \frac{2x^2}{9} - \frac{14x^3}{81} + \frac{35x^4}{243} - \frac{91x^5}{729}$$

$$+ \frac{728x^6}{6561} - \frac{1976x^7}{19683} + \dots$$

(i.) 1.069178\dots; (iii.) 1.0455159\dots

(ii.) 1.000667555\dots;

$$141. (1-x)^{\frac{1}{2}} = 1 - \frac{2x}{5} - \frac{3x^2}{25} - \frac{8x^3}{125} - \frac{26x^4}{625} - \frac{468x^5}{15625}$$

$$- \frac{1794x^6}{78125} - \dots = 0.998398075\dots \text{ when } x = 0.004.$$

$$142. (1+15)^{\frac{1}{2}} = 15^{\frac{1}{2}} \left( 1 + \frac{1}{4 \times 15} - \frac{3}{32 \times 15^2} + \frac{7}{128 \times 15^3} \right.$$

$$\left. - \frac{77}{2048 \times 15^4} + \frac{231}{8192 \times 15^5} - \dots \right).$$

143.  $27^{\frac{1}{3}} \left(1 + \frac{2}{27}\right)^{\frac{1}{3}} = 27^{\frac{1}{3}} \left(1 + \frac{2}{3 \times 27} - \frac{2^2}{3^2 \times 27^2} + \frac{5 \times 2^3}{3^4 \times 27^3} - \frac{10 \times 2^4}{3^5 \times 27^4} + \frac{22 \times 2^5}{3^6 \times 27^5} - \frac{154 \times 2^6}{3^8 \times 27^8} + \dots\right).$
144.  $9^{-\frac{1}{2}} \left(1 + \frac{1}{9}\right)^{-\frac{1}{2}} = \frac{1}{3} \left(1 - \frac{2}{9} + \frac{1}{27} - \frac{4}{9^2} + \frac{5}{9^3} - \frac{6}{9^4} + \frac{7}{9^5} - \frac{8}{9^6} + \dots\right).$
145.  $64^{\frac{1}{4}} \left(1 + \frac{1}{64}\right)^{\frac{1}{4}} = 8 \left(1 + \frac{1}{2 \times 64} - \frac{1}{2^3 \times 64^2} + \frac{1}{2^4 \times 64^3} - \frac{5}{2^7 \times 64^4} + \frac{7}{2^8 \times 64^5} - \frac{21}{2^{10} \times 64^6} + \dots\right).$
146.  $a^{-5} - 5a^{-6}x + 15a^{-7}x^2 - 35a^{-8}x^3 + 70a^{-9}x^4 - 126a^{-10}x^5 + 210a^{-11}x^6 - 330a^{-12}x^7 + \dots$
147.  $a^{-8} + 6a^{-7}x + 21a^{-6}x^2 + 56a^{-5}x^3 + 126a^{-4}x^4 + 252a^{-3}x^5 + 462a^{-2}x^6 + 792a^{-1}x^7 + \dots$
148.  $\frac{1}{64b^6} + \frac{3y}{64b^7} + \frac{21y^2}{256b^8} + \frac{7y^3}{64b^9} + \frac{63y^4}{512b^{10}} + \frac{63y^5}{512b^{11}} + \frac{231y^6}{2048b^{12}} + \frac{99y^7}{1024b^{13}} + \dots$
149.  $\frac{512}{c^9} - \frac{9212z}{c^{10}} + \frac{92160z^2}{c^{11}} - \frac{675840z^3}{c^{12}} + \frac{4055040z^4}{c^{13}} - \frac{21086208z^5}{c^{14}} + \frac{98402304z^6}{c^{15}} - \frac{421724160z^7}{c^{16}} + \dots$
150.  $\frac{1}{6561} + \frac{8x}{19683} + \frac{4x^2}{6561} + \frac{40x^3}{59049} + \frac{110x^4}{177147} + \frac{88x^5}{177147} + \frac{572x^6}{1594323} + \frac{1144x^7}{4782969} + \dots$

$$151. \frac{1}{a^9} - \frac{9x}{2a^{10}} + \frac{45x^2}{4a^{11}} - \frac{165x^3}{8a^{12}} + \frac{495x^4}{16a^{13}} - \frac{1287x^5}{32a^{14}} + \frac{3003x^6}{64a^{15}} \\ - \frac{6435x^7}{128a^{16}} + \dots$$

$$152. 1 - 10x + 55x^2 - 220x^3 + 715x^4 - 2002x^5 + 5005x^6 \\ - 11,440x^7 + \dots$$

$$153. 1 + 11y + 66y^2 + 286y^3 + 1001y^4 + 3003y^5 + 8008y^6 \\ + 19,448y^7 + \dots$$

$$154. \frac{1}{z^9} - \frac{9}{z^{10}} + \frac{45}{z^{11}} - \frac{165}{z^{12}} + \frac{495}{z^{13}} - \frac{1287}{z^{14}} + \frac{3003}{z^{15}} - \frac{6435}{z^{16}} + \dots$$

$$155. 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots$$

$$156. \frac{1}{a^{10}} - \frac{5x^2}{a^{12}} + \frac{15x^4}{a^{14}} - \frac{35x^6}{a^{16}} + \frac{70x^{12}}{a^{18}} - \frac{126x^{15}}{a^{20}} + \frac{210x^{18}}{a^{22}} \\ - \frac{330x^{21}}{a^{24}} + \dots$$

$$157. \frac{1}{b^{24}} + \frac{8x^2}{b^{27}} + \frac{36x^4}{b^{30}} + \frac{120x^6}{b^{33}} + \frac{330x^8}{b^{36}} + \frac{792x^{10}}{b^{39}} + \frac{1716x^{12}}{b^{42}} \\ + \frac{3432x^{14}}{b^{45}} + \dots$$

$$158. \frac{1}{c^9} - \frac{18x^2}{c^{10}} + \frac{180x^4}{c^{11}} - \frac{1320x^6}{c^{12}} + \frac{7920x^{12}}{c^{13}} - \frac{41184x^{15}}{c^{14}} \\ + \frac{192192x^{18}}{c^{15}} - \frac{823680x^{21}}{c^{16}} + \dots$$

$$159. \frac{1}{a^{13}c^6} - \frac{6x^2}{a^{13}c^7} + \frac{21x^4}{a^{14}c^8} - \frac{56x^6}{a^{15}c^9} + \frac{126x^8}{a^{16}c^{10}} - \frac{252x^{10}}{a^{17}c^{11}} + \frac{462x^{12}}{a^{18}c^{12}} \\ - \frac{792x^{14}}{a^{19}c^{13}} + \dots$$

$$160. \frac{1}{a^3} + \frac{6x^2}{a^3\sqrt{a}} + \frac{21x^4}{a^4} + \frac{56x^6}{a^4\sqrt{a}} + \frac{126x^8}{a^5} + \frac{252x^{10}}{a^5\sqrt{a}} \\ + \frac{462x^{12}}{a^6} + \frac{792x^{14}}{a^6\sqrt{a}} + \dots$$

$$161. \sqrt{b} + \frac{h}{2\sqrt{b}} - \frac{h^2}{8b\sqrt{b}} + \frac{h^3}{16b^2\sqrt{b}} - \frac{5h^4}{128b^3\sqrt{b}} \\ + \frac{7h^5}{256b^4\sqrt{b}} - \frac{21h^6}{1024b^5\sqrt{b}} + \frac{33h^7}{2048b^6\sqrt{b}} - \dots$$

$$162. \sqrt[4]{b} - \frac{x}{4\sqrt[4]{b^3}} - \frac{3x^2}{32b\sqrt[4]{b^3}} - \frac{7x^3}{128b^2\sqrt[4]{b^3}} - \frac{77x^4}{2048b^3\sqrt[4]{b^3}} \\ - \frac{231x^5}{8192b^4\sqrt[4]{b^3}} - \frac{1463x^6}{65536b^5\sqrt[4]{b^3}} - \frac{4807x^7}{262144b^6\sqrt[4]{b^3}} - \dots$$

$$163. \sqrt[5]{2a} + \frac{y}{5\sqrt[5]{16a^4}} - \frac{y^2}{25a\sqrt[5]{16a^4}} + \frac{3y^3}{250a^2\sqrt[5]{16a^4}} \\ - \frac{21y^4}{5000a^3\sqrt[5]{16a^4}} + \frac{399y^5}{250000a^4\sqrt[5]{16a^4}} \\ - \frac{399y^6}{625000a^5\sqrt[5]{16a^4}} + \frac{1653y^7}{6250000a^6\sqrt[5]{16a^4}}$$

$$164. \sqrt[6]{2} + \frac{a}{6\sqrt[6]{2^5}} - \frac{5a^2}{144\sqrt[6]{2^5}} + \frac{55a^3}{5184\sqrt[6]{2^5}} - \frac{935a^4}{497664\sqrt[6]{2^5}} \\ + \frac{4301a^5}{2985984\sqrt[6]{2^5}} - \frac{124729a^6}{429981696\sqrt[6]{2^5}} \\ + \frac{623645a^7}{5159780352\sqrt[6]{2^5}} - \dots$$

$$165. x + \frac{a}{2x} - \frac{a^2}{8x^3} + \frac{a^3}{16x^5} - \frac{5a^4}{128x^7} + \frac{7a^5}{256x^9} - \frac{21a^6}{1024x^{11}} \\ + \frac{33a^7}{2048x^{13}} - \dots$$

$$166. a - \frac{1}{2a} - \frac{1}{8a^3} - \frac{1}{16a^5} - \frac{5}{128a^7} - \frac{7}{256a^9} - \frac{21}{1024a^{11}} \\ - \frac{33}{2048a^{13}} - \dots$$

$$167. \sqrt[7]{a} + \frac{2b}{7\sqrt[7]{a^6}} - \frac{12b^2}{49a\sqrt[7]{a^5}} + \frac{104b^3}{343a^2\sqrt[7]{a^4}} - \frac{1040b^4}{2401a^3\sqrt[7]{a^3}} \\ + \frac{11232b^5}{16807a^4\sqrt[7]{a^2}} - \frac{127296b^6}{117649a^5\sqrt[7]{a}} + \frac{10438272b^7}{5764801a^6\sqrt[7]{a}} - \dots$$

$$168. 1 + \frac{a}{9} - \frac{4a^2}{81} + \frac{68a^3}{2187} - \frac{442a^4}{19683} + \frac{3094a^5}{177147} - \frac{68068a^6}{4782969} \\ - \frac{515372a^7}{43046721} - \dots$$

$$169. 1 + \frac{y}{10} - \frac{9y^2}{200} + \frac{57y^3}{2000} - \frac{1653y^4}{80000} + \frac{64467y^5}{4000000} \\ - \frac{1052961y^6}{80000000} + \frac{8874957y^7}{800000000} - \dots$$

$$170. 1 + \frac{z^2}{9} - \frac{4z^4}{81} + \frac{68z^6}{2187} - \frac{442z^8}{19683} + \frac{3094z^{10}}{177147} - \frac{68068z^{12}}{4782969} \\ + \frac{515372z^{14}}{43046721} - \dots$$

$$171. 1 + \frac{3h}{4} - \frac{3h^2}{32} + \frac{5h^3}{128} - \frac{45h^4}{2048} + \frac{117h^5}{8192} - \frac{663h^6}{65536} \\ + \frac{1989h^7}{262144} - \dots$$

$$172. \sqrt[7]{2^4} - \frac{4y^3}{7\sqrt[7]{2^3}} - \frac{3y^5}{49\sqrt[7]{2^5}} - \frac{5y^7}{343\sqrt[7]{2^7}} - \frac{85y^{11}}{19208\sqrt[7]{2^8}} \\ - \frac{51y^{15}}{33614\sqrt[7]{2^9}} - \frac{527y^{18}}{941192\sqrt[7]{2^{10}}} - \frac{10013y^{21}}{46118408\sqrt[7]{2^{11}}} - \dots$$

$$\begin{aligned}
 173. \quad & \sqrt[5]{3^3} + \frac{3d^4}{5\sqrt[5]{3^3}} - \frac{d^8}{25\sqrt[5]{3^3}} + \frac{7d^{12}}{1125\sqrt[5]{3^3}} - \frac{7d^{16}}{5625\sqrt[5]{3^3}} \\
 & + \frac{357d^{20}}{1265625\sqrt[5]{3^3}} - \frac{1309d^{24}}{18984375\sqrt[5]{3^3}} \\
 & + \frac{187d^{28}}{284765625\sqrt[5]{3^3}} - \dots
 \end{aligned}$$

$$\begin{aligned}
 174. \quad & a^2\sqrt[5]{a^2} - \frac{4b}{5\sqrt[5]{a^3}} - \frac{2b^2}{25a^3\sqrt[5]{a^3}} - \frac{4b^3}{125a^6\sqrt[5]{a^3}} - \frac{11b^4}{625a^9\sqrt[5]{a^3}} \\
 & - \frac{176b^5}{15625a^{12}\sqrt[5]{a^3}} - \frac{616b^6}{78125a^{15}\sqrt[5]{a^3}} \\
 & - \frac{2288b^7}{390625a^{18}\sqrt[5]{a^3}} - \dots
 \end{aligned}$$

$$\begin{aligned}
 175. \quad & y\sqrt[4]{y^3} - \frac{7b^3}{8\sqrt[4]{y}} - \frac{7b^6}{128y^3\sqrt[4]{y}} - \frac{21b^9}{1024y^6\sqrt[4]{y}} \\
 & - \frac{357b^{12}}{32768y^9\sqrt[4]{y}} - \frac{1785b^{15}}{262144y^{12}\sqrt[4]{y}} - \frac{19635b^{18}}{4194304y^{15}\sqrt[4]{y}} \\
 & - \frac{115005b^{21}}{33554432y^{18}\sqrt[4]{y}} - \dots
 \end{aligned}$$

$$\begin{aligned}
 176. \quad & \frac{1}{a\sqrt{a}} - \frac{1}{2a^4\sqrt{a}} + \frac{3}{8a^7\sqrt{a}} - \frac{5}{16a^{10}\sqrt{a}} + \frac{35}{128a^{13}\sqrt{a}} \\
 & - \frac{63}{256a^{16}\sqrt{a}} + \frac{231}{1024a^{19}\sqrt{a}} - \frac{429}{2048a^{22}\sqrt{a}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 177. \quad & \frac{1}{x} + \frac{a}{2x^3} + \frac{3a^2}{8x^5} + \frac{5a^3}{16x^7} + \frac{35a^4}{128x^9} + \frac{63a^5}{256x^{11}} + \frac{231a^6}{1024x^{13}} \\
 & + \frac{429a^7}{2048x^{15}} + \dots
 \end{aligned}$$

$$178. 1 + \frac{x^5}{3} + \frac{2x^{10}}{9} + \frac{14x^{15}}{81} + \frac{35x^{20}}{243} + \frac{91x^{25}}{729} + \frac{728x^{30}}{6561} \\ + \frac{1976x^{35}}{19683} + \dots$$

$$179. 1 - \frac{2d}{5} + \frac{12d^2}{25} - \frac{88d^3}{125} + \frac{704d^4}{625} - \frac{29568d^5}{15625} \\ + \frac{256256d^6}{78125} - \frac{2269696d^7}{390625} + \dots$$

$$180. \frac{1}{8} - \frac{3h}{256} + \frac{3h^2}{2048} - \frac{13h^3}{65536} + \frac{117h^4}{4194304} - \frac{2691h^5}{671088640} \\ + \frac{6279h^6}{10737418240} - \frac{29601h^7}{343597383680} + \dots$$

$$181. \frac{1}{27} + \frac{x^2}{81} + \frac{5x^4}{1458} + \frac{35x^6}{39366} + \frac{35x^8}{157464} + \frac{77x^{10}}{1417176} \\ + \frac{1001x^{12}}{76527504} + \frac{715x^{14}}{229582512} + \dots$$

$$182. \frac{1}{a^5} - \frac{15y^4}{7a^{12}} + \frac{270y^8}{49a^{19}} - \frac{5130y^{12}}{343a^{26}} + \frac{100035y^{16}}{2401a^{33}} \\ - \frac{1980693y^{20}}{16807a^{40}} + \frac{39613860y^{24}}{117649a^{47}} - \frac{5585554260y^{28}}{5764801a^{54}} + \dots$$

$$183. \sqrt[4]{a} - \frac{\sqrt[12]{a^9x^4}}{2b} + \frac{3a^{12}\sqrt[12]{a^3x^8}}{8b^2} - \frac{5ax\sqrt[4]{a^3}}{16b^3} + \frac{35a^2x^{12}\sqrt[12]{a^3x^4}}{128b^4} \\ - \frac{63a^2x^{12}\sqrt[12]{a^9x^8}}{256b^5} + \frac{231a^3x^2\sqrt[4]{a}}{1024b^6} - \frac{429a^3x^{12}\sqrt[12]{a^9x^4}}{2048b^7} + \dots$$

$$184. \frac{\sqrt[3]{x}}{\sqrt[9]{a^3}} - \frac{x^2\sqrt[3]{x}}{3a\sqrt[9]{a^3}} + \frac{2x^8\sqrt[3]{x}}{9a^3\sqrt[9]{a^3}} - \frac{14x^9\sqrt[3]{x}}{81a^5\sqrt[9]{a^3}} + \frac{35x^{12}\sqrt[3]{x}}{243a^8\sqrt[9]{a^3}} \\ - \frac{91x^{15}\sqrt[3]{x}}{729a^{10}\sqrt[9]{a^3}} + \frac{728x^{18}\sqrt[3]{x}}{6561a^{10}\sqrt[9]{a^3}} - \frac{1976x^{21}\sqrt[3]{x}}{19683a^{11}\sqrt[9]{a^3}} + \dots$$

- 
185.  $2a\sqrt{2a} + \frac{24a^2\sqrt{2a}}{x^2} + \frac{240a^3\sqrt{2a}}{x^4} + \frac{2240a^4\sqrt{2a}}{x^6}$   
 $+ \frac{20160a^5\sqrt{2a}}{x^8} + \frac{177408a^6\sqrt{2a}}{x^{10}} + \frac{1537536a^7\sqrt{2a}}{x^{12}}$   
 $+ \frac{13178880a^8\sqrt{2a}}{x^{14}} + \dots$
186.  $\frac{1}{\sqrt[4]{8}} - \frac{3}{8}\sqrt[4]{\frac{x^3}{4}} + \frac{21x}{128}\sqrt[4]{\frac{x^3}{2}} - \frac{77x^2\sqrt[4]{x}}{1024} + \frac{1155x^3}{16384\sqrt[4]{8}}$   
 $- \frac{4389x^3\sqrt[4]{x^3}}{131072} + \frac{33649x^4\sqrt[4]{x^3}}{2097152} - \frac{129789x^5\sqrt[4]{x}}{16777216} + \dots$
187.  $\frac{1}{a^5} - \frac{3x^3}{a^8} + \frac{6x^6}{a^{10}} - \frac{10x^9}{a^{12}} + \frac{15x^{12}}{a^{14}} - \frac{21x^{15}}{a^{16}} + \frac{28x^{18}}{a^{18}}$   
 $- \frac{36x^{21}}{a^{20}} + \dots$
188.  $\frac{1}{a^2} + \frac{2x^3}{3a^5} + \frac{5x^6}{9a^8} + \frac{40x^9}{81a^{11}} + \frac{110x^{12}}{243a^{14}} + \frac{308x^{15}}{729a^{17}} + \frac{2618x^{18}}{6561a^{20}}$   
 $+ \frac{7480x^{21}}{19683a^{23}} + \dots$
189.  $a\sqrt[4]{a^3} - \frac{7a^{12}\sqrt[3]{ab^4}}{2} + \frac{35^{12}\sqrt[3]{a^9b^3}}{8} - \frac{35b^4\sqrt[4]{a}}{16} + \frac{35b^3\sqrt[3]{b}}{128\sqrt[4]{a}}$   
 $+ \frac{7b^3\sqrt[3]{b^3}}{256\sqrt[4]{a^3}} + \frac{7b^3}{1024a\sqrt[4]{a}} + \frac{5b^3\sqrt[3]{b}}{2048a\sqrt[4]{a^3}} + \dots$
190.  $\frac{1}{a} - \frac{p}{a^2} + \frac{p^2}{a^3} - \frac{p^3}{a^4} + \frac{p^4}{a^5} - \frac{p^5}{a^6} + \frac{p^6}{a^7} - \frac{p^7}{a^8} + \dots$
191.  $\frac{1}{b^6} - \frac{3r}{b^8} + \frac{6r^2}{b^{10}} - \frac{10r^3}{b^{12}} + \frac{15r^4}{b^{14}} - \frac{21r^5}{b^{16}} + \frac{28r^6}{b^{18}} - \frac{36r^7}{b^{20}} + \dots$



$$192. -\left(\frac{1}{H^3} + \frac{5t}{H^3} + \frac{15t^2}{H^3} + \frac{35t^3}{H^3} + \frac{70t^4}{H^3} + \frac{126t^5}{H^3} + \frac{210t^6}{H^3} + \frac{330t^7}{H^3} + \dots\right).$$

$$193. \frac{1}{Y^4} + \frac{4v}{Y^3} + \frac{10v^2}{Y^3} + \frac{20v^3}{Y^3} + \frac{35v^4}{Y^3} + \frac{56v^5}{Y^3} + \frac{84v^6}{Y^3} + \frac{120v^7}{Y^3} + \dots$$

$$194. -\left(\frac{1}{\sqrt[3]{4a^3}} + \frac{1}{4a^3\sqrt[3]{4a^3}} + \frac{1}{8a^4\sqrt[3]{4a^3}} + \frac{7}{96a^5\sqrt[3]{4a^3}} + \frac{35}{768a^6\sqrt[3]{4a^3}} + \frac{91}{3072a^7\sqrt[3]{4a^3}} + \frac{91}{4608a^8\sqrt[3]{4a^3}} + \frac{247}{18432a^9\sqrt[3]{4a^3}} + \dots\right).$$

$$195. \frac{1}{\sqrt{x^3}} - \frac{2}{7x^3\sqrt{x^3}} + \frac{16}{49x^{10}\sqrt{x^3}} - \frac{160}{343x^{18}\sqrt{x^3}} + \frac{1760}{2401x^{20}\sqrt{x^3}} - \frac{20416}{16807x^{25}\sqrt{x^3}} + \frac{244992}{117649x^{30}\sqrt{x^3}} - \frac{21069312}{5764801x^{35}\sqrt{x^3}} + \dots$$

$$196. \frac{1}{\sqrt[15]{81^3x^3}} + \frac{8\sqrt[4]{a}}{15x\sqrt[15]{81x}} + \frac{8\sqrt{a}}{25x\sqrt[15]{81^3x^{13}}} + \frac{224\sqrt[4]{a^3}}{1125x^2\sqrt[15]{81^3x^3}} + \frac{2128a}{16875x^3\sqrt[15]{81x}} + \frac{34048a\sqrt[4]{a}}{421875x^3\sqrt[15]{81^3x^{13}}} + \frac{987392a\sqrt{a}}{18984375x^4\sqrt[15]{81^3x^3}} + \frac{9591808a\sqrt[4]{a^3}}{284765625x^5\sqrt[15]{81x}} + \dots$$

$$\begin{aligned}
 197. \quad & \frac{1}{\sqrt[5]{a^4y^3}} - \frac{2\sqrt[3]{x}}{5ay\sqrt[5]{a^4y^3}} + \frac{7\sqrt[3]{x^2}}{25a^2y^2\sqrt[5]{a^4y^3}} - \frac{28x}{125a^3y^3\sqrt[5]{a^4y^3}} \\
 & + \frac{119x\sqrt[3]{x}}{625a^4y^4\sqrt[5]{a^4y^3}} - \frac{2618x\sqrt[3]{x^2}}{15625a^5y^5\sqrt[5]{a^4y^3}} \\
 & + \frac{11781x^2}{78125a^6y^6\sqrt[5]{a^4y^3}} - \frac{53856x^2\sqrt[3]{x}}{390625a^7y^7\sqrt[5]{a^4y^3}} + \dots
 \end{aligned}$$

$$\begin{aligned}
 198. \quad & z^2\sqrt[5]{81z} + \frac{4\sqrt[5]{h}}{5\sqrt[5]{3z^4}} - \frac{2\sqrt[5]{h^2}}{75z^4\sqrt[5]{3z^4}} + \frac{4\sqrt[5]{h^3}}{1125z^8\sqrt[5]{3z^4}} \\
 & - \frac{11\sqrt[5]{h^4}}{16875z^{12}\sqrt[5]{3z^4}} + \frac{176h}{1265625z^{16}\sqrt[5]{3z^4}} \\
 & - \frac{616h\sqrt[5]{h}}{18984375z^{20}\sqrt[5]{3z^4}} + \frac{2288h\sqrt[5]{h^2}}{284765625z^{24}\sqrt[5]{3z^4}} - \dots
 \end{aligned}$$

199. 3.1622776 .....

211. 3.87827796 .....

200. 7.2801098 .....

212. 2.7686217 .....

201. 4.431047 .....

213. 5.02897093 .....

202. 3.107232 .....

214. 6.00303358 .....

203. 2.8716219 .....

215. 2.02174421 .....

204. 3.0385047 .....

216. 18.566355 .....

205. 4.073062 .....

217. 4.99899969 .....

206. 5.99988978 .....

218. 3.00045703 .....

207. 0.30983874 .....

219. 1.028901 .....

208. 0.954640 .....

220. 2.00002083 .....

209. 5.87367008 .....

221. 6.8463237 .....

210. 4.33012703 .....

222. 4.083680 .....

## Ex. 58. Page 201.

1.  $\frac{1}{2}$ ;  $\frac{2}{3}$ .
2. The number of cases favorable to the event being represented by  $m$ , and the number of cases unfavorable by  $n$ ; then  $m : n$  are the odds in favor of the event, and the probability of its happening is represented by the fraction  $\frac{m}{m+n}$ . (i.) If  $m$  is 0, the chance of drawing a white ball is  $\frac{0}{10}$ , or zero; that is, the bag contained no white ball. (ii.) If  $n$  is equal to 0, the bag contained 10 white and no black; therefore, the chance of drawing a white ball is  $\frac{10}{10}$ , or unity; that is, certainty. (iii.) If  $n = m$ ,  $P$  becomes  $\frac{m}{2m} = \frac{1}{2}$ ; and one of the five white balls is as likely to be drawn as one of the black ones.
3.  $5 \begin{cases} 4 \text{ black,} \\ 1 \text{ white.} \end{cases}$
4.  $\frac{1}{5}$ .
5.  $\frac{1}{6}$ ;  $\frac{1}{18}$ ;  $\frac{1}{12}$ ;  $\frac{1}{36}$ ;  $\frac{5}{12}$ .
6. 11 : 19.
7. 1 : 1.
8.  $\frac{116}{117}$ ,  $\frac{117}{117}$ .
9.  $\frac{2}{11}$ ,  $\frac{5}{33}$ ,  $\frac{12}{33}$ .
10.  $\frac{1}{12}$ .
11.  $\frac{8}{11}$ .
12.  $\frac{1}{2}$ ;  $\frac{3}{14}$ ;  $\frac{1}{5}$ ;  $\frac{8}{35}$ ;  $\frac{28}{70}$ ;  $\frac{33}{35}$ .
13. 17 : 2 against.
14. The number of ways in which they could come 1, 3, 5, ....., is

$$8 + \frac{8 \times 7 \times 6}{1 \times 2 \times 3} + \frac{8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5} + \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} = 128.$$

The number of ways in which they would come 2, 4, 6, ....., is

$$\frac{8 \times 7}{1 \times 2} + \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} + \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = 126.$$

Therefore, the ratio is  $128 : 126 = 64 : 63$ . Similarly, for 5.

15.  $\frac{51}{595}$ . 16.  $\frac{100}{857}$ ;  $\frac{10}{714}$ . 17.  $\frac{1}{105}$ ;  $\frac{27}{1331}$ . 18.  $\frac{14}{11}$ ;  $\frac{336}{1331}$ .

19. 5; 11. 20. Chance for each =  $\frac{1}{11}$ .

21. Because the second and third terms of  $(\frac{1}{2} + \frac{1}{2})^5$  are alike.

22.  $\frac{125}{216}$ ;  $\frac{1}{216}$ ;  $\frac{75}{216}$ . 23.  $\frac{17}{11}$ . 24.  $7\frac{1}{2}$ .

25. The general total of the chances is 216, and either 10 or 11 may be thrown in 27 different ways. The odds then are 27 : 189 or 1 : 7.

26.  $\frac{3}{20}$ ;  $\frac{1}{10}$ .

Ex. 59. Page 204.

1.  $4 + \frac{1}{1 + \frac{1}{9 + \frac{1}{2}}}$

3.  $7 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}$

5.  $\frac{1}{2 + \frac{1}{2 + \frac{1}{33}}}$

2.  $3 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3}}}$

4.  $20 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4}}}$

6.  $\frac{1}{6 + \frac{1}{4 + \frac{1}{8 + \frac{1}{3}}}}$

7.  $\frac{1}{5 + \frac{1}{2 + \frac{1}{3 + \frac{1}{21}}}}$

8.  $7 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$

$$9. \quad 1 + \frac{1}{16 + \frac{1}{1 + \frac{1}{12 + \frac{1}{2}}}}$$

$$14. \quad 5 + \frac{1}{14 + \frac{1}{3 + \frac{1}{2}}}$$

$$10. \quad 3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{14}}}}}$$

$$15. \quad \frac{1}{41 + \frac{1}{2 + \frac{1}{39 + \frac{1}{1 + \frac{1}{2}}}}}$$

$$11. \quad \frac{1}{5 + \frac{1}{2 + \frac{1}{3 + \frac{1}{7 + \frac{1}{1 + \frac{1}{2}}}}}}$$

$$16. \quad \frac{1}{a + \frac{1}{3a + \frac{1}{5a}}}$$

$$12. \quad \frac{1}{3 + \frac{1}{5 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{5}}}}}}$$

$$17. \quad \frac{1}{5m + \frac{1}{7m^2 + \frac{1}{9m^3}}}$$

$$18. \quad \frac{1}{1 + \frac{1}{x + \frac{1}{1 + \frac{1}{x^2}}}}$$

$$13. \quad 9 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{61 + \frac{1}{1 + \frac{1}{3}}}}}}$$

$$19. \quad \frac{1}{x + \frac{1}{2x + \frac{1}{3x + \frac{1}{4x}}}}$$

$$20. \quad x + \frac{1}{x^2 + \frac{1}{x^2 + \frac{1}{x^4}}}$$

$$21. \quad \frac{1}{a^2x + \frac{1}{a^2x^2 + \frac{1}{ax^3 + \frac{1}{x^4}}}}$$

22.  $\frac{2}{3}$ .

25.  $\frac{4}{5}$ .

28.  $3\frac{3}{4}$ .

31.  $5\frac{7}{8}$ .

23.  $\frac{2}{5}$ .

26.  $2\frac{26}{115}$ .

29.  $7\frac{21}{180}$ .

32.  $\frac{2655}{47844}$ .

24.  $\frac{12}{179}$ .

27.  $\frac{242}{188}$ .

30.  $1\frac{1}{16}$ .

33.  $\frac{2307}{47844}$ .

34.  $\frac{1}{2}$ ;  $\frac{2}{3}$ ;  $\frac{13}{167}$ ;  $\frac{88}{167}$ ;  $\frac{431}{167}$ .

36.  $\frac{1}{2}$ ;  $\frac{7}{15}$ ;  $\frac{22}{15}$ ;  $\frac{205}{15}$ ;  $\frac{842}{15}$ .

35.  $\frac{1}{8}$ ;  $\frac{1}{4}$ ;  $\frac{5}{18}$ ;  $\frac{6}{18}$ ;  $\frac{185}{184}$ .

37.  $\frac{1}{4}$ ;  $\frac{5}{8}$ ;  $\frac{122}{88}$ ;  $\frac{106}{1067}$ ;  $\frac{1198}{4920}$ .

38.  $\frac{1}{4}$ ;  $\frac{2}{5}$ ;  $\frac{19}{111}$ ;  $\frac{115}{111}$ ;  $\frac{1061}{111}$ ;  $\frac{11676}{111}$ .

39.  $\frac{1}{2}$ ;  $\frac{2}{3}$ ;  $\frac{5}{12}$ ;  $\frac{13}{12}$ ;  $\frac{78}{12}$ ;  $\frac{229}{12}$ .

40.  $\frac{2}{3}$ ;  $\frac{1}{4}$ ;  $\frac{22}{27}$ ;  $\frac{27}{27}$ ;  $\frac{241}{27}$ ;  $\frac{268}{27}$ .

41.  $\frac{1}{4}$ ;  $\frac{1}{5}$ ;  $\frac{2}{14}$ ;  $\frac{4}{19}$ ;  $\frac{51}{242}$ ;  $\frac{463}{2197}$ ;  $\frac{277}{4888}$ .

42.  $\frac{12}{12}$ ;  $\frac{12}{12}$ ;  $\frac{27}{27}$ ;  $\frac{22}{27}$ ;  $\frac{502}{27}$ ;  $\frac{525}{27}$ ;  $\frac{2882}{15652}$ ;  $\frac{6252}{6252}$ ;  $\frac{15600}{15600}$ .

43.  $\frac{1}{4}$ ;  $\frac{2}{5}$ ;  $\frac{5}{4}$ ;  $\frac{2}{7}$ ;  $\frac{167}{180}$ ;  $\frac{179}{180}$ ;  $\frac{243}{180}$ ;  $\frac{892}{180}$ ;  $\frac{2731}{180}$ .

44.  $\frac{1}{4}$ ;  $\frac{2}{5}$ ;  $\frac{2}{4}$ ;  $\frac{5}{7}$ ;  $\frac{2}{11}$ ;  $\frac{12}{11}$ ;  $\frac{21}{11}$ ;  $\frac{24}{11}$ ;  $\frac{428}{11}$ ;  $\frac{1237}{11}$ ;  $\frac{6165}{11}$ .

45.  $\frac{1}{4}$ ;  $\frac{5}{8}$ ;  $\frac{11}{18}$ ;  $\frac{16}{18}$ ;  $\frac{157}{18}$ ;  $\frac{577}{18}$ ;  $\frac{2425}{18}$ ;  $\frac{12557}{18}$ ;  $\frac{13052}{18}$ ;  $\frac{26621}{18}$ .

46.  $\frac{1}{4}$ ;  $\frac{4}{5}$ ;  $\frac{8}{9}$ ;  $\frac{11}{9}$ ;  $\frac{28}{27}$ ;  $\frac{27}{27}$ ;  $\frac{228}{27}$ ;  $\frac{262}{27}$ ;  $\frac{842}{27}$ ;  $\frac{1022}{27}$ ;  $\frac{2708}{27}$ .

47.  $\frac{1}{2}$ ;  $\frac{6}{18}$ ;  $\frac{817}{1770}$ ;  $\frac{2157}{1770}$ ;  $\frac{2274}{1770}$ ;  $\frac{5731}{17418}$ ;  $\frac{29427}{17418}$ .

48.  $\frac{1}{11}$ ;  $\frac{1}{12}$ ;  $\frac{2}{36}$ ;  $\frac{7}{36}$ ;  $\frac{10}{117}$ ;  $\frac{57}{667}$ ;  $\frac{181}{2118}$ ;  $\frac{262}{11267}$ ;  $\frac{1148}{18875}$ ;  $\frac{5524}{64757}$ ; etc.

49.  $\frac{1}{4}$ ;  $\frac{17}{16}$ ;  $\frac{29}{16}$ ;  $\frac{102}{16}$ ;  $\frac{225}{16}$ ;  $\frac{428}{16}$ ;  $\frac{1998}{16}$ ;  $\frac{4279}{16}$ ;  $\frac{14717}{16}$ .

50.  $\frac{2}{3}$ ;  $\frac{2}{3}$ ;  $\frac{218}{27}$ ;  $\frac{245}{27}$ ;  $\frac{29828}{27}$ ;  $\frac{29828}{27}$ ;  $\frac{172551}{55248}$ .

51.  $\frac{2}{3}$ ;  $\frac{2}{3}$ ;  $\frac{8}{9}$ ;  $\frac{11}{4}$ ;  $\frac{12}{7}$ ;  $\frac{82}{89}$ ;  $\frac{106}{71}$ ;  $\frac{1264}{465}$ ;  $\frac{1457}{465}$ ; etc.

52.  $\frac{1}{2}$ ;  $\frac{2}{3}$ ;  $\frac{19}{18}$ ;  $\frac{28}{18}$ ;  $\frac{48}{18}$ ;  $\frac{76}{178}$ ;  $\frac{271}{1618}$ ;  $\frac{1278}{1618}$ ; etc.

53.  $\frac{1}{4}$ ;  $\frac{1}{4}$ ;  $\frac{17}{16}$ ;  $\frac{21}{16}$ ;  $\frac{269}{16}$ ;  $\frac{277}{16}$ ;  $\frac{1725}{16}$ .

$$54. 3 + \frac{1}{3} + \frac{1}{6} + \dots$$

$$58. 6 + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \dots$$

$$55. 3 + \frac{1}{1} + \frac{1}{6} + \dots$$

$$59. 7 + \frac{1}{7} + \frac{1}{14} + \dots$$

$$56. 4 + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \dots$$

$$60. 6 + \frac{1}{4} + \frac{1}{12} + \dots$$

$$57. 5 + \frac{1}{5} + \frac{1}{10} + \dots$$

$$61. 8 + \frac{1}{1} + \frac{1}{16} + \dots$$

$$62. \frac{\sqrt{29}-5}{2}, 64. \frac{\sqrt{13}-3}{2}, 66. \frac{\sqrt{5}-1}{2}, 68. \frac{\sqrt{15}-3}{2}.$$

$$63. \sqrt{5}-2, 65. \sqrt{3}, 67. \frac{3-\sqrt{5}}{2}.$$

$$69. \frac{\text{meter}}{\text{yard}} = \frac{1}{1}; \frac{11}{10}; \frac{12}{11}; \frac{35}{32}; \frac{292}{267}; \frac{5875}{5372}; \frac{6167}{5639}.$$

$$70. \frac{\text{fathom}}{\text{meter}} = \frac{1}{1}; \frac{2}{1}; \frac{9}{5}; \frac{11}{6}; \frac{64}{35}; \frac{267}{146}.$$

$$71. \frac{20 \text{ dollars}}{20 \text{ francs}} = \frac{5}{1}; \frac{26}{5}; \frac{57}{11}; \frac{482}{93}; \frac{539}{104}; \frac{1021}{197}; \frac{6665}{1286}.$$

$$72. \frac{\text{eq. diameter}}{\text{pol. diameter}} = \frac{1}{1}; \frac{299}{298}; \frac{1795}{1789}; \frac{3889}{3876}; \frac{13462}{13417}.$$

$$73. \frac{\text{tropical year}}{\text{civil year}} = \frac{1}{1}; \frac{1507}{1506}; \frac{1508}{1507}; \frac{3015}{3013}; \frac{4523}{4520};$$

$$\frac{7538}{7533}; \frac{12061}{12053}.$$

Ex. 60. Page 208.

$$1. 1 - 6x + 24x^2 - 96x^3. \quad 2. \frac{1}{8} + \frac{1}{8}x + \frac{1}{24}x^2 + \dots$$

$$3. \frac{1}{8} + \frac{1}{8}a + \frac{1}{24}a^2 + \frac{1}{81}a^3 + \frac{1}{216}a^4 + \dots$$

$$4. \frac{1}{2} - \frac{5}{4}x + \frac{15}{8}x^2 - \frac{45}{16}x^3.$$

$$5. 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$6. 1 - 2x + 3x^2 - 5x^3 + 8x^4 - \dots$$

$$7. 1 + 3x + 7x^2 + 17x^3 + 41x^4 + \dots$$

$$8. 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 - \dots$$

$$9. \frac{a}{b} + \frac{ac}{b^2}x + \frac{ac^2}{b^3}x^2 + \frac{ac^3}{b^4}x^3 + \dots$$

$$10. a + \frac{b^2}{2a} - \frac{b^4}{8a^3} + \frac{b^6}{16a^5}.$$

$$11. \frac{1}{2x} + \frac{3}{4} + \frac{9x}{8} + \frac{27x^2}{16} + \frac{81x^3}{32} + \dots$$

$$12. 1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3 + \frac{3}{128}x^4 - \dots$$

$$13. \frac{x}{m} + \frac{x^2}{m^2} - \frac{x^4}{m^4} - \frac{x^5}{m^5} + \dots$$

$$14. 1 - \frac{3}{x} + \frac{5}{x^2} - \frac{7}{x^3} + \dots$$

$$15. \frac{1}{3(x+2)} + \frac{5}{3(x-1)}.$$

$$16. \frac{2}{3+4x} + \frac{5}{6+7x}.$$

$$17. \frac{1}{2(x+1)} - \frac{1}{x+2} + \frac{1}{2(x+3)}.$$

$$18. \frac{9}{8x-7} + \frac{6}{5x-4} + \frac{3}{2x-1}.$$

$$19. \frac{Pa - Q}{(a-b)(1+ax)} - \frac{Pb - Q}{(a-b)(1+bx)}.$$

$$20. \frac{\frac{bc-ad}{cf-de}}{c-dx} + \frac{\frac{af-be}{cf-de}}{e-fx}.$$



$$21. \frac{1}{(x-3)} + \frac{3}{(x+2)}.$$

$$22. \frac{3}{2(x-1)} - \frac{7}{x-2} + \frac{13}{2(x-3)}.$$

$$23. \frac{1}{x^3} + \frac{1}{x} + \frac{1}{(x-1)^2} - \frac{1}{x-1}.$$

$$24. \frac{1}{3(x+1)} + \frac{5x-4}{3(x^2-x+1)}.$$

$$25. -\frac{2}{x^3} + \frac{3}{x} - \frac{3x-2}{(x^2+1)^2} - \frac{3x-2}{x^2+1}.$$

$$26. \frac{1}{ax} - \frac{1}{a(x-a)} + \frac{2}{(x-a)^2}.$$

Ex. 61. Page 210.

- |  |  |
|--|--|
| 1. 24, 24, 24.....   | 13. 10.414439.                         |
| 2. 0.  | 14. $\frac{1}{x(x+2)}$ ; $\frac{3}{4}$ |
| 3. 4; 256.   | 15. $\frac{1}{\frac{x}{2}(x+1)}$ ; 2.  |
| 4. 169; $n^2$ .  | 16. $\frac{11}{18}$ .                  |
| 5. $n(n+1)$ ; 156.   | 17. $\frac{1}{4}$ .                    |
| 6. $\frac{1}{48}$ .  | 18. $\frac{3}{8}$ .                    |
| 7. 385; $\frac{n}{3} + \frac{n^2}{2} + \frac{n}{6}$ .                | 19. $\frac{n}{12+12n}$ .               |
| 8. 3150.   | 20. $\frac{1+4x+x^2}{(1-x)^4}$ .       |
| 9. 364; $\frac{n}{1} \times \frac{n+1}{3} \times \frac{n+2}{3}$ .    | 21. $\frac{18}{140}$ .                 |
| 10. $\frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$ . | 22. 4760.                              |
| 11. $\log 512 = 2.709269$ .  |  |
| 12. 3.6342411.   |  |

23.  $\frac{1}{18}$ .

25.  $\frac{1+2x-x^2}{1-2x+x^2-2x^2}$ .

24.  $\frac{n}{3}(4n^2-1)$ .

26.  $\frac{1+2x+3x^2}{(1-x)^2}$ ; 6146.

## Ex. 62. Page 212.

1.  $x^3-6x^2+11x-6=0$ . 4.  $x^3+8x^2-5x-84=0$ .

2.  $x^3-3x+2=0$ .

5.  $x^3-7x^2-5x+35=0$ .

3.  $x^3-19x+30=0$ .

6.  $x^3-2x^2+3x-6=0$ .

7.  $x^3-8x-3=0$ .

8.  $6x^4-35x^3+62x^2-35x+6=0$ .

9.  $x^4+x^3-26x^2-44x+48=0$ .

10.  $2x^4-6x^3+3x^2-1=0$ . 23. 256,  $\frac{32}{18}$ .

11.  $\pm 3, \pm 2$ .

24.  $\sqrt[3]{\frac{9}{16}}, \sqrt{-\frac{1}{2}}$ .

12.  $\pm 5, 2\sqrt{-1}$ .

25. 1, -4.

13.  $\sqrt{15}, \sqrt{-14}$ .

26. 4, 1,  $\frac{-5 \pm \sqrt{-11}}{2}$ .

14. 0, 1, 8.

27.  $\pm 3\sqrt{\pm 1}, \pm 2\sqrt{\pm 1}$ .

15. 9, 5, 3, -1.

16. 0, 1,  $\frac{1}{2}(1+\sqrt{22})$ .

28. 9,  $\sqrt[3]{16}$ .

17.  $\frac{3}{2}, \sqrt{-\frac{12}{4}}$ .

29. 1, 4, 2, 3.

18.  $\sqrt[3]{3}, \sqrt{-\frac{3}{4}}$ .

30. 5, 3,  $4 \pm \sqrt{10}$ .

19. 5, -4 $\frac{2}{3}$ .

31. 4,  $\sqrt[3]{\frac{1}{4}}$ .

20.  $\frac{1}{4}, -\frac{3}{8}$ .

32. 2, -4,  $-1 \pm \sqrt{\frac{47}{2}}$ .

21.  $\frac{1}{8}, -42\frac{1}{8}$ .

33. 0, 3,  $\frac{5}{2}, -2\frac{3}{2}$ .

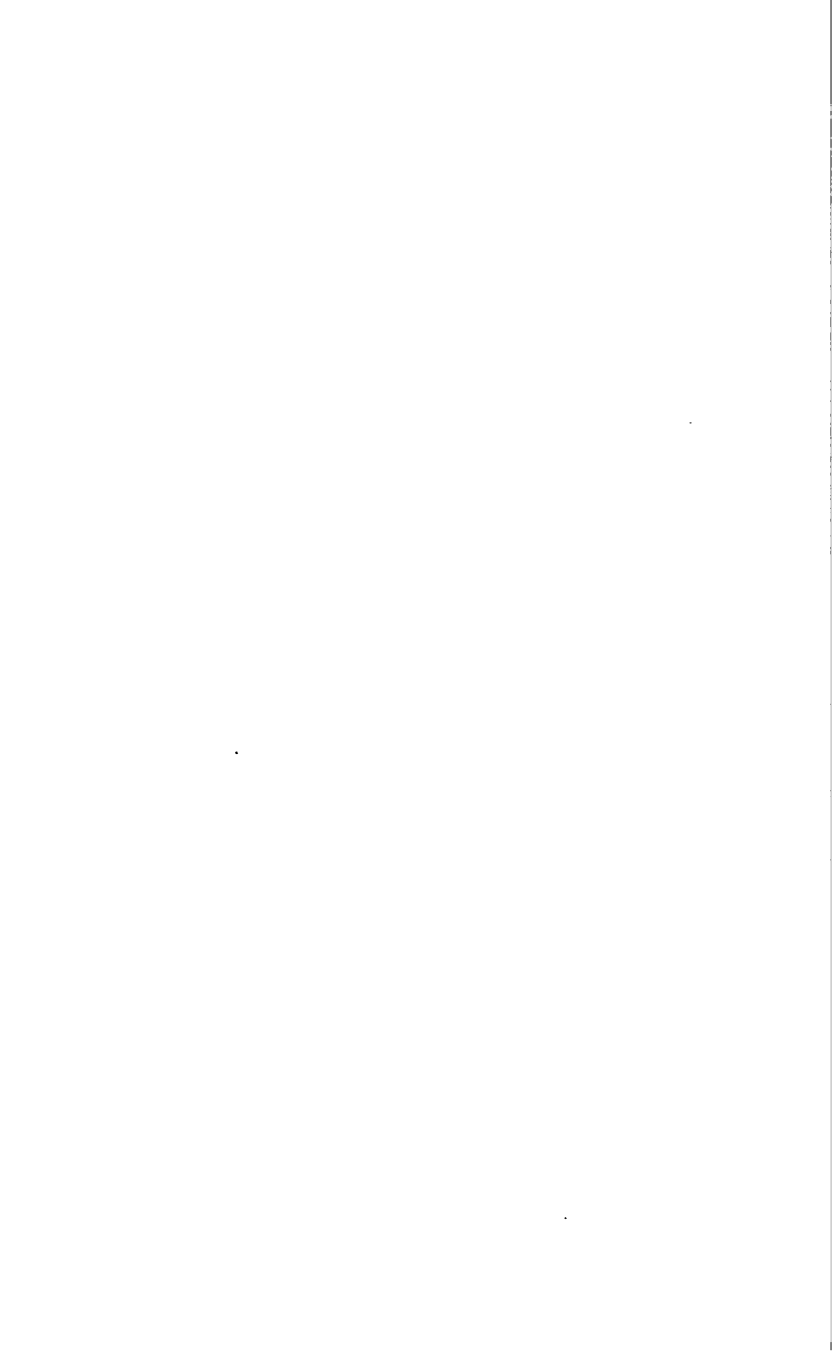
22. 2, -3.

34. 4,  $\sqrt[3]{25}$ .

35. 7,  $-\frac{7}{2}$ .  
 36. 4, 1,  $\frac{-3 \pm \sqrt{-7}}{2}$ .  
 37.  $\pm \sqrt{c(2a-c)}$ .  
 38.  $\frac{4}{3}$ ,  $-\frac{1}{3}$ ,  $\frac{9 \pm \sqrt{481}}{50}$ .  
 39. 2, -5,  $\frac{1}{2}(3 \pm \sqrt{-7})$ .  
 40. 9, 4.  
 41. 4,  $-2 \pm \sqrt{-22}$ .  
 42. 2, 3, 4.  
 43. 3,  $\frac{-3 \pm \sqrt{-7}}{2}$ .  
 44. -1, 2, 7.  
 45. 3,  $-\frac{3}{2} \pm \frac{\sqrt{45}}{2}$ .  
 46. 2, 5, 11.  
 47. -3,  $\frac{3 \pm \sqrt{-7}}{2}$ .  
 48. 1, 2, 3, 4.  
 49. 5,  $\frac{-5 \pm \sqrt{-83}}{2}$ .  
 50. 1, -2, -6, 7.  
 51. 1,  $-\frac{1}{2}(1 \pm \sqrt{-3})$ .  
 52. 1,  $1\frac{1}{2}$ , 2.  
 53.  $\frac{1}{5}$ ,  $-\frac{2}{10}$ ,  $-\frac{3}{5}$ .  
 54.  $-\frac{4}{3}$ ,  $\frac{2 \pm \sqrt{-127}}{3}$ .  
 55.  $\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}$ .  
 56. 0.32748.  
 57. 3.135.  
 58. -0.6378.  
 59. 7.6172.  
 60. 1.42535.  
 61.  $y^3 - 7y = 90$ .  
 62.  $y^3 - 100 = 0$ .  
 63.  $y^3 + 6y - 88 = 0$ .  
 64.  $y^3 + 6y - 2 = 0$ .  
 65.  $y^3 - \frac{31}{2}y - \frac{349}{2} = 0$ .  
 66.  $y^4 - \frac{17}{2}y^2 + \frac{225}{16} = 0$ .  
 67. 4,  $1 \pm \sqrt{-1}$ .  
 68. 1.1541,  
 $-0.57708 \pm 1.9997\sqrt{-1}$ .  
 69. -4,  $5 \pm \sqrt{-3}$ .  
 70. -2, -5, -5.  
 71. 10,  $\frac{11}{2} \pm \frac{5}{2}\sqrt{-3}$ .  
 72. 3,  $\frac{1}{3}$ ,  $\frac{1}{3}(\pm\sqrt{55} - 8)$ .  
 73.  $\frac{1}{2}(1 \pm \sqrt{-3})$ ,  $\frac{1}{2}(2 \pm 0)$ ,  
 $\frac{1}{2}(3 \pm \sqrt{5})$ .  
 74. 1, 1.56155, -2.56155.  
 75. 2.18479, 2.22668,  
 $-3.41147$ .  
 76. 11.00675.  
 77.  $-\frac{1}{2}$ ,  $-\frac{2}{5}$ ,  $-\frac{3}{7}$ ,  $-\frac{5}{12}$ ,  $-\frac{23}{35}$ ,  
 $-\frac{129}{287}$ ,  $-\frac{143}{242}$ .  
 78. 4, -2.

## Ex. 63. Page 215.

- |   |                                  |               |
|---|----------------------------------|---------------|
| 1. 5.   | 5. 2.718281.                     | 9. - 8.       |
| 2. $x = 1.19897$ .  | 6. $x = 5$ .                     | 10. 0.355207. |
| 3. 0.4342945.   | 7. - 0.991318.....               | 11. 7.        |
| 4. 3.86633.   | 8. $x = 2\frac{1}{2}$ .          |               |
| 12. $x = \frac{m \log b - n \log a}{m \log a - n \log b}$ .   | 13. $x = -1.553174$ .            |               |
|   | 14. $x = 3; y = 5$ .             |               |
| 15. $x = \frac{\log b \log q - \log d \log p}{\log b \log c - \log a \log d};$<br>$y = \frac{\log c \log p - \log a \log q}{\log b \log c - \log a \log d}$ |                                  |               |
| 16. $x = 3; y = 4$ .  | 21. $x = 3; y = 5$ .             |               |
| 17. $x = \frac{\log b \pm \sqrt{b^2 - 4} - \log 2}{\log a}$ .   | 22. $x = 7; y = 121$ .           |               |
|   | 23. $x = 2; y = 3; z = 2$ .      |               |
| 18. $x = 3.597$ .   | 24. $x = 7$ or 4.                |               |
| 19. 3.2164.   | 25. $x = 10$ or 1; $x = 0.003$ . |               |
| 20. 1.38736.  | 26. $x = 7$ or - 12.             |               |



## PART SECOND.

### ANSWERS TO EXAMINATION MANUAL.



#### 1.

- |                        |                         |
|------------------------|-------------------------|
| 1. 94.                 | 5. $xy(x^2 - y^2)$ .    |
| 2. $10a - b + 3c$ .    | 6. $\frac{1}{2}$ .      |
| 3. $1 - x^5$ .         | 7. 9.                   |
| 4. $xy(a^2x - b^2y)$ . | 8. 2 inches; 10 inches. |

#### 2.

- |                      |                          |
|----------------------|--------------------------|
| 1. -4.               | 5. $\frac{2(a-c)}{ac}$ . |
| 2. $a + 2b - c$ .    | 6. 1.                    |
| 3. $a^2(x-y)(x+y)$ . | 7. A \$2400; B \$2500.   |
| 4. $3x + 5$ .        | 8. $x = 5, y = 4$ .      |

#### 3.

- |                           |                     |                               |
|---------------------------|---------------------|-------------------------------|
| 1. 2.                     | 4. $2a^2(a-x)$ .    | 7. A 36 years;<br>B 18 years. |
| 2. $x^4 - \frac{a^4}{16}$ | 5. $\frac{a+x}{ax}$ | 8. $x = 2, y = 1$ .           |
| 3. $2a - \frac{1}{2}x$ .  | 6. 7.               |                               |

#### 4.

- |   |                               |
|---|-------------------------------|
| 1. $a^2 - b^2 + 2bc - c^2$ ; 80.                      | 5. $x = 13, y = 7$ .          |
| 2. $a + 9c + b - 14d - 6e$ .                          | 6. $(y+30)(y-5)$ .            |
| 3. $x^5 + x^4y + x^3y^2 + x^2y^3$<br>$+ xy^4 + y^5$ . | 7. $\frac{(a-2)(a-5)}{a^2}$ . |
| 4. 72.  | 8. 420.                       |

5.

1.  $3xy - 3bx + 3ay - 2ab - ac + ad.$
2.  $4x^2(x-3).$
3.  $12a^2b^2(a^2-b^2)^2.$
4.  $\frac{x^2-4}{x^2-9}$
5.  $\frac{1}{2}.$
6.  $\$26\frac{3}{4}.$
7. 2h. 8m.  $34\frac{3}{4}$ s.

6.

1.  $a + b + c.$
2.  $2a^3 - 11a^2b - 2ab^2 + 2b^3 + 8.$
3.  $x^5 - x^2y^2 + x^2y^2 - y^5.$
4.  $a^2 + ab + b^2 + ac - bc + c^2.$
5. 4.
6.  $\frac{2x^2+1}{x^2-1}$
7.  $x=3, y=2, z=1.$
8.  $\$900.$

7.

1.  $40\frac{3}{4}.$
2.  $x.$
3.  $1 + 6x + 16x^2 + 28x^3 + 27x^4 + 18x^5.$
4.  $x^4 + 2x^2y - 2xy^2 - y^4.$
5.  $x^2 + (b+c)x + bc.$
6.  $\frac{a+b}{a-b}.$
7. 8.
8. 20 years.

8.

1.  $1\frac{1}{2}.$
2. 60.
3.  $xy^2 - x - x^2 + y^2.$
4.  $mpx^2 + (mq - np)x^2 - (mr + nq)x + nr.$
5.  $1 + x^2 + 4y^2 - x + 2y + 2xy.$
6. 1.
7. 320 yards by 180 yards.

9.

1. 27.
2.  $x^2a - 2x^2y^2 + 2y^2b.$
3.  $a(x-1)(x+1);$   
 $(x^2+9a^2)(x+3a)(x-3a);$   
 $(2a+3)(4a^2-6a+9);$   
 $(x+5)(x+2).$
4.  $(a^2-b^2)(b^2+c^2)(c^2-a^2)(b-c).$
5.  $x - y + z.$
6.  $-x - 2y + 4.$
7.  $\frac{a(6a-b)}{6b-a}.$
8.  $46\frac{1}{2}$  and  $23\frac{1}{2}.$

## 10.

1. 25.
2.  $3x^3 - 3x^2 + 3x - 3$ .
3.  $2a^{3n} - 2x^{3n}$ .
4.  $x^2 - 3x - y$ .
5.  $(x^3 + y^3)(x^4 + y^4)(x^2 + y^2)(x + y)(x - y);$   
 $\left(2x^3 + \frac{1}{x}\right)\left(4x^4 - 2x + \frac{1}{x^3}\right);$   
 $(4x^6 + y^2)(2x^3 + y)(2x^3 - y);$   
 $(x+7)(x-1).$
6.  $2a + 3x$ .
7.  $\frac{2ab}{a+b}$ .
8. A 40 years; B 20 years.

## 11.

1. 10.
2.  $2x^4 + \frac{1}{2}x^2 + \frac{3}{2}x + \frac{1}{2}$ .
3.  $1 + 2x + 3x^2 + 4x^3 + 5x^4$ .
4.  $-6x^2$ .
5.  $(2x - 5)(x + 1);$   
 $(a + b - c)(a - b - c).$
6.  $y^2 + my + m^2$ .
7. 7.
8. \$3.75.

## 12.

1.  $3a + b$ .
2.  $-x - 2y - 4$ .
3.  $3a^2 - 2ab + 5b^2$ .
4. 66 years.
5.  $x^4 - \frac{9x^2y^2}{16} + \frac{3xy^3}{2} - y^4$ .
6. 5.
7.  $\frac{3}{16}$ .
8.  $x = 7, y = 8, z = 9$ .

## 13.

1.  $11x^3 - 5x + 4y$ .
2.  $2x + 6a + 13b - 7c - 13$ .
3.  $27x^3 - 36x^2 + 6xy + 24x$   
 $- y^3 - 4y^2 - 8y - 8$ .
4.  $x - 2$ .
5. 13.
6.  $(z + x - y)(z - x + y)$ .
7.  $\frac{x+y}{y}$ .
8.  $x = 1, y = -1, z = 2$ .



14.

1.  $x^2 + x - \frac{1}{2}$ .

2.  $x^2 - 2$ .

3.  $(x+7)(x+13)$ .

4. 52, 34.

5.  $\frac{b}{a-b}$

6.  $\frac{a^2(x^2 - a^2)}{x^2(x^2 + a^2)}$

7.  $x = 35, y = 30, z = 25$ .

15.

1.  $a^5 - 4a^4b + 4a^3b^2 + 4a^2b^3 - 17ab^4 - 12b^5$ .

2.  $x^2 - ax + b^2$ .

3.  $\frac{2a}{b} + \frac{2b}{a}$ .

4.  $2\frac{1}{3}$ .

5.  $x = 4, y = 0, z = 5$ .

6.  $(x-z-a+y)(x-z+a-y)$ .

7. 0.

16.

1.  $a - b$  feet; north; south.

2.  $8x - 8$ .

3.  $a + 2b - c$ .

4.  $a^4(a - 3b^2)(a + 3b^2)$ .

5.  $2(x + a)$ .

6.  $\frac{y}{x+y}$

7. 3.    8. \$3, \$5, and \$7.

17.

1.  $(a+b-c)(a-b+c)(a+b+c)(a-b-c)$ .

2.  $x(2x-y)$ .

3.  $5b - 5c$ .

4.  $x^4 - x^2y^2 + y^4$ .

5.  $x = y = 5$ .

6. 49, 42.

7.  $4x^2 - 4x + 1$ .

18.

1.  $(a-b+c)x^2 + (a+b-c)xy - (a-b-c)y^2$ .

2.  $x^4 - (a^2 + b^2)x^2 + a^2b^2$ .

3.  $(2x+y)(16x^4 - 8x^2y + 4x^2y^2 - 2xy^3 + y^4);$   
 $(5x-7y)(x+3y); (3ax+1)(2ax-3)$ .

4. a.    5.  $30x(1-x)^2(1+x)$ .    6.  $x-1$ .    7.  $2\frac{1}{3}$ .    8. 24 days.

## 19.

1.  $ax^{m+n} + ax^m - (a-b)x^{m-n} + ax^{m-2n}$ .
2.  $-(3ab + b^2)$ .
3.  $\frac{a+b+c}{a-b+c}$ .
4. 0.
5.  $a - 3b$ .
6.  $\frac{4a^2 - 3ab - b^2 + 4a}{4a + b}$ .
7.  $\frac{(a-2b)(2a-b)}{a}$ .
8. 64 square yards.

## 20.

1.  $2a^{2n} - 4a^n b^n + 2b^{2n}$ .
2.  $x(5x^2 - 1)$ .
3.  $-1$ .
4.  $(2x-1)(2x+1)(4x^2+1)$   
 $(16x^4+1)$  and  $(x-9)(x+2)$ .
5. 1.
6.  $\frac{ac}{b}$ .
7. 28.

## 21.

1.  $(a-x)^2(a+x); (x-2y)(x+2y)(x+3y)$ .
2.  $\frac{1-x}{x(2x-1)(2x+1)}$ .
3.  $\frac{a+b}{ab(a-b)}$ .
4. 1.
5.  $8x^2 - 14x + 6$ .
6.  $7+x$ .
7. \$240.

## 22.

1.  $(a^2x^2 + \frac{2}{3}y^2)(ax + \frac{2}{3}y)(ax - \frac{2}{3}y);$   
 $bx(3x-by)(9x^2 + 3bxy + b^2y^2);$   
 $2a(5x-3a)(2x+3a)$ .
2.  $\frac{x^2+1}{x^2-1}$ .
3.  $(x+2y)(x-2y)(x+3y)$ .
4. 7.
5. 10.
6. 24 days.

## 23.

1.  $-1$ .
2.  $\frac{7}{8}a - \frac{1}{8}b + \frac{1}{4}c - \frac{1}{12}d$ .
3.  $x^4 + 2ax^3 + 3a^2x^2 + 2a^3x + a^4$ .
4.  $\frac{x^2 - ax + a^2}{x + a}$ .
5.  $x = \frac{2}{3}, y = \frac{2}{3}$ .
6.  $x^2 - x + \frac{1}{4}$ .
7. After 8 hours; 38 miles from the starting-place.

## 24.

- |                     |                     |
|---------------------|---------------------|
| 1. 42.              | 4. $4x(x-y)$ .      |
| 2. $x^2 - 6x + 9$ . | 5. $x = 7, y = 5$ . |
| 3. $2ab$ .          | 6. 104.             |

## 25.

- |  |  |
|--|--|
| 1. $(x^4 - x^2y^2 + y^4)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^2 + y^2)(x+y)(x-y)$ . |  |
| 2. $a^3 - 3b^3 + 3c^3$ .   | 5. $\frac{4a-b}{2a-3c}$                        |
| 3. $2x^2(2x+3a)$ .   | 6. $x = 1, y = \frac{1}{8}, z = \frac{1}{8}$ . |
| 4. $\frac{a+c}{a-c}$   | 7. 75.   |

## 26.

- |                                  |                             |
|----------------------------------|-----------------------------|
| 1. $x = 4, y = 6, z = 8$ .       | 5. $a^3 - 2ax + bx - x^3$ . |
| 2. $2\frac{1}{2}$ miles an hour. | 6. $\frac{2x}{x+y}$         |
| 3. $x^2 - 2x + 1$ .              | 4. 21.                      |

## 27.

- |   |                            |
|---|----------------------------|
| 1. $2xy^2 - (3x^2 - a)y^2 - (x^3 - a^3)y + 2a^2x^2$ . | 4. $\frac{x}{x^2 + y^2}$   |
| 2. $\frac{a^2 - 4b^2}{a}$                             | 5. 25 and 26.              |
| 3. $a + b$ .  | 6. $134\frac{1}{8}$ miles. |
|   | 7. $x = 24, y = 12$ .      |

## 28.

- |   |                               |
|---|-------------------------------|
| 1. $x^3 + x^5 + x^7; \frac{x^9}{1-x^2}$ . | 5. $\frac{x+a}{2a}$           |
| 2. $6x^4 - (3a^2 + 12)x^2 - 4ax + 3a^3$ . | 6. A 62 h.; B 93 h.; C 155 h. |
| 3. $a - y$ .                              | 7. $x = 5, y = 2$ .           |

## 29.

1.  $\frac{a^2 + x^2}{2ax}$
2.  $\frac{1}{1 - x + x^2}$
3.  $x = 3, y = 4.$
4.  $x(x^2 - 2x + 1).$
5.  $x, x - 2y, x + 2y, x(x - 2y),$   
 $x(x + 2y), (x - 2y)(x + 2y),$   
 $x(x - 2y)(x + 2y).$
6. 52.
7. 8.

## 30.

2.  $a^2 + b^2.$
3.  $\frac{a^2}{x} - \frac{x^2}{a^2}.$
5.  $3a - b.$
4.  $\frac{a}{b}.$
6.  $x = 1, y = 3, z = 5.$
7. 35; 14.

## 31.

2.  $x = \frac{m^2 - n^2}{ma - nb}, y = \frac{n^2 - m^2}{na - mb}.$
3.  $x = 2, y = 3, z = 4.$
4.  $x + 3.$
5.  $\frac{(x + y)(x^2 + xy + y^2)}{x \cdot y}.$
6.  $\frac{3}{4}.$
7.  $(1 + b + a^2)x^3 - (c^2 + c + 1)x^2 - (a + b + c)x.$

## 32.

2.  $2(2x^2 + 7x + 3).$
3. 1.
4.  $\frac{ac}{b}.$
5.  $x = 10, y = 6.$
6.  $x = 7, y = 9, z = 11.$
7. 588 and 708.

## 33.

1.  $-\frac{2a^2}{b^2 + 2b}.$
2.  $x = \frac{ac + d}{b + c}, y = \frac{ab - d}{b + c}.$
3.  $x = 5, y = 4, z = 3.$
4. 0.
5.  $\frac{7x}{6a}.$
6.  $x^4 + 2ax^3 + 3a^2x^2 + 2a^3x + a^4.$
7.  $-2b(3a + b)(a^2 - 2b^2).$
8. 25; 15.

34.

$$2. 4b^2(a+x)^2;$$

$$216a^2b^4x^3(a^2-x^2)^2.$$

$$3. a^{10} - 1.$$

$$4. \frac{2+2x+x^2}{3+5x+3x^2+x^3}.$$

$$5. \frac{a^3+b^3}{a^4b^2+a^2b^4}.$$

$$6. x=6, y=4.$$

$$7. \$140, \$60, \$70, \$20.$$

$$8. 32x^{10}y^5z^{15}, -343a^9b^{12}c^{15},$$

$$81a^4b^{16}c^8.$$

35.

$$1. 0.$$

$$2. \frac{a+b}{a-b}.$$

$$3. x=y=z=12.$$

$$4. 144a(a^2-9).$$

$$5. x^2-4x-21.$$

$$6. 10 \text{ P.M. ; half way.}$$

36.

$$1. 0.$$

$$2. ax^2-bxy+cy^2.$$

$$3. a-b.$$

$$4. (a^2+x^2)(a^2+x^2)(a^2-x^2).$$

$$5. \frac{a+2}{2}.$$

$$6. A's \text{ age } 45; B's \text{ age } 20.$$

$$8. \frac{a^2}{b^3} + 3\frac{a}{b} + 3\frac{b}{a} + \frac{b^3}{a^3}.$$

37.

$$1. 1.$$

$$2. \frac{ab(a+b)^2}{a^2+ab+b^2}.$$

$$3. x=4, y=5, z=6.$$

$$4. 24\frac{1}{2} \text{ by } 12\frac{1}{2}.$$

$$5. x^2 - \frac{1}{2}x + \frac{1}{2}.$$

$$6. \frac{x^2}{y^3} - 3\frac{x^2}{y^3} + 3 - \frac{y^2}{x^2}.$$

38.

$$1. \frac{8x^3}{a^3} - \frac{27y^3}{b^3}.$$

$$2. x+3.$$

$$3. \frac{14x+4}{x^2-1}.$$

$$4. x^2 + \frac{1}{8}x + \frac{1}{8}.$$

$$5. \frac{a}{a-b}.$$

$$6. A \$3700; B \$3300.$$

$$7. 8a^3 - 36\frac{a^2}{x} + 54\frac{a}{x^2} - \frac{27}{x^3}.$$

## 39.

1.  $\frac{1}{abc}$
2. 24.
3.  $x=8, y=12$ .
4.  $\frac{a}{3b} - \frac{3b}{a}$
5.  $8x-8$ .
6.  $bc(a^2-1)$ .
7. A 116; B 109; C 114.

## 40.

1.  $3b(2a^2-a^2b-2ab^2+b^3)$ .
2.  $3a+b$ .
3.  $8(1-x)^2(1+x)(1+x^2)$ .
4.  $1-2x+3x^2$ .
5.  $\frac{1}{4}$ .
6.  $x=6, y=3, z=8$ .
7.  $43\frac{7}{11}$  min. past 2 o'clock.

## 41.

1.  $1-\frac{1}{2}xy-2x^2y^2$ .
2.  $\frac{1+5a}{3(1+2a)}$ .
3.  $x=3, y=2, z=1$ .
4.  $\frac{x^2}{3} - \frac{17x^2}{36} + \frac{x}{3} - \frac{1}{8}$ .
5.  $2\frac{2}{15}$  miles from one end;  
 $1\frac{4}{15}$  miles from the other.
6.  $\frac{a^3}{8} + \frac{a^2b}{2} + \frac{2ab^2}{3} + \frac{8b^3}{27}$ .
7.  $3a$ .

## 42.

1.  $\frac{18}{x^2-9}$
2.  $x-y$ .
3.  $a-\frac{b}{3}$
4.  $\frac{3}{4}$ .
5.  $x=1\frac{1}{2}, y=\frac{1}{2}, z=1\frac{7}{8}$ .
6.  $2+4\sqrt{a}$ .
7.  $13\frac{1}{2}$  inches.

## 43.

1.  $(9x^2-x-3)(x+6)(x+5)$ .
2.  $x^2-2x-\frac{3}{2}$ .
3.  $x=10, y=8, z=6$ .
4.  $\frac{5}{2}x^2+3ax-\frac{7}{2}a^2$ .
5.  $\frac{4}{3(x+1)}$ .
6.  $\frac{yz+xz+xy}{yz+xz-xy}$ .
7.  $\frac{x^2-x}{2x-1}$ .

## 44.

1.  $\frac{2}{(2x+1)(2x-1)^2}$ .
2.  $\frac{16ax}{(a+x)^2}$ .
4.  $(x^2+y^2)(x+y)(x-y)$ .
5.  $a+2b+c$ .
3.  $\frac{5a^2}{12} + \frac{5b^2}{12} + \frac{5c^2}{12}$ .
6.  $a+b$ .
7. 18, 22, 10, 40.

## 45.

1.  $\frac{2}{(1+a)(1-4a^2)}$ .
5. -1.
2.  $-1+3x^2-x^4$ .
6.  $\frac{a^2b^3}{64} - \frac{3ab}{16} + \frac{3}{4ab} - \frac{1}{a^2b^3}$ .
3.  $x+2$ .
7.  $x=\frac{1}{3}, y=\frac{1}{4}, z=\frac{1}{5}$ .
4.  $(1+x)(1-x)^2(1+x^2)$ .

## 46.

1.  $\frac{ax^2-ax}{a^2x-a^2-x^2}$ .
4.  $\frac{4a^2-3b}{8a+3ac-3}$ .
2.  $\frac{x-3}{2x^2+5}$ .
5. Father's age, 50; son's age, 25.
3.  $\frac{2x^2-3xy-2y^2}{4xy}$ .
6.  $x=17, y=19$ .
7.  $\frac{2x(1+ax^2)}{2+ax-2x^2}$ .

## 47.

1.  $a^{10}-1$ .
4.  $x=2, y=1, z=3$ .
2.  $2(2x^2+7x+3)$ .
5. \$3\frac{1}{2}.
3.  $(x^3-1)(x^3+1)(x+1)(x-1)$ .
6. 42.

## 48.

1.  $\frac{(x+1)^2}{2}$ .
4.  $\frac{2x+1}{5x+3}$ .
5.  $\frac{x+2}{x+3}$ .
2.  $x-3$ .
6.  $\frac{a+m^2}{b+n^2}; 0$ .
3.  $\frac{5x}{(x+1)(2x-1)(3x+1)}$ .

## 49.

1.  $c$ .

2.  $x = \frac{cb' - bc'}{ab' - ba'}, y = \frac{ca' - ac'}{a'b - ab'}$ .

3.  $x = 5, y = -5, z = 5$ .

4.  $x = \frac{2}{m+p-n}, y = \frac{2}{m+n-p}, z = \frac{2}{n+p-m}$ .

5.  $x = 12, y = 30, z = 168$ .

## 50.

1. After 8 hours; 38 miles from the starting-place.

2. 860. 3. A \$8; B \$4.

4. 2 hours 24 minutes;  $\frac{mnp}{mn+mp+np}$  hours.

## 51.

1.  $(a-b+c)x^2 - (b-c+d)x^2 - (c+d+e)x$ .

2.  $a^2 + b^2 + c^2 + ab - ac + bc$ . 4. 69; 81.

3.  $\frac{a+bx}{b+ax}$  5.  $\pm \frac{1}{2}$ .

6. 3, or -1.

## 52.

1.  $2b(x+y)$ . 3.  $3(ax+2)$ . 5. 3, or  $-\frac{1}{2}$ .

2. 22. 4.  $\pm \frac{a}{8}$ . 6. 40 yds.; 24 yds.

## 53.

1.  $24a^2b^2(a^2-b^2)$ . 4.  $8x^{\frac{1}{2}} + 4x^{\frac{1}{2}}y^{\frac{1}{2}} + 2x^{\frac{1}{2}}y + y^{\frac{3}{2}}$ .

2.  $\frac{2a^3}{a^4-x^4}$ . 3.  $\frac{1}{32c^{\frac{1}{2}}}$ . 5. 7, or -2.

6. 25; 20.



## 54.

1.  $x = -5$ ,  $y = 6$ ,  $z = -2$ .
2. \$180; \$150.
3.  $\frac{a}{b^3}$ .
4.  $x^{-3} + x^{-3}y^{-3} + y^{-3}$
5.  $\sqrt[5]{\frac{1}{18}}$ .
6. 5, or  $-4\frac{1}{8}$ .

## 55.

1.  $36xy^2(x^2 - y^2)$ .
2.  $\frac{x - 3x^2 + 3x^3}{(1 - x)^3}$ .
3.  $3\sqrt[3]{7}$ ;  $4\sqrt{2}$ ;  $2\sqrt[3]{22}$ .
4.  $x^2 - ax - a^2$ .
5.  $18\sqrt[3]{2}$ .
6. 1, or  $10\frac{1}{8}$ .

## 56.

2.  $2ax\sqrt[6]{72ab^5}$ .
3.  $13\sqrt[3]{2}$ .
4. 1.
5.  $2\frac{1}{2}$ , or  $-4\frac{7}{10}$ .
6. 358 yards.

## 57.

1.  $x^{-4}y^4$ .
2.  $\sqrt{2}$ .
3. \$60, or \$40.
4.  $-4$ , or  $-21$ .
5.  $x = 4$ ,  $y = -5$ ,  $z = 6$ .
6. 1540; 880; 616.

## 58.

1. 13.
2.  $a(a^2 - 1)(a + 3)(a - 6)$ .
3. 147 miles.
4.  $\frac{1}{8}\sqrt[4]{27}$ .
5. 6, or 3.
6. A \$1800; B \$1600.

## 59.

1.  $x = \frac{ac + b^2}{a^2 + b}$ ,  $y = \frac{ab - c}{a^2 + b}$ .
2.  $-\frac{7}{4}$ .
3.  $x^2 - 2$ .
4.  $\frac{2y^3}{x(x^2 - y^2)}$ .
5.  $a - 2x + 3$ .
6.  $54\frac{6}{11}$  minutes past 10.
7. 4, or  $-1$ .

## 60.

1. 8.
2.  $\frac{(x+y)(x^2+xy+y^2)}{y}$
3.  $x^2+7$ .
4.  $x+\frac{x}{y}+1$ .
5. 12.
6.  $\frac{7}{4}$ .
7. 2 miles an hour.

## 61.

1. 2, or  $-\frac{3}{4}$ .
2.  $\frac{2}{a}$
3.  $x=a$ ,  $y=b$ .
4.  $12x-5$ .
5.  $a^2+b^2+c^2-ab-ac-bc$ .
6.  $x^2-x+\frac{1}{4}$ .
7. The first pipe 6 minutes, and the second pipe 4 minutes.

## 62.

1.  $6a^2+25ab-13ac-b^2+c^2$ ; 100.
2.  $x^4+x^2+1+\frac{1}{x^2}+\frac{1}{x^4}$
3.  $\frac{(m+n)^2}{2n}$
4.  $4x-1$ .
5.  $a-2b^2-3c^3$ .
6.  $x=5$ ,  $y=6$ .
7. 1, or  $\frac{35}{81}$ .
8.  $32\frac{8}{11}$  minutes past 3.

## 63.

1.  $3\frac{1}{2}$ .
2.  $x=3$ ,  $y=2$ ,  $z=1$ .
3. 1, or  $\frac{23}{7}$ .
4.  $x^4-7x^2+26x-40$ .
5.  $b$ .
6. \$125.

## 64.

1.  $\frac{18x^2+26x-14}{2(x+2)(2x+1)}$
2.  $\frac{3}{4}x^3-\frac{1}{2}x^2+1$ .
3.  $(a^2-b)(a^2+b)(a-b)$ .
4.  $a+\frac{b}{2a}-\frac{b^2}{8a^3}$ , etc.
5.  $\frac{a+b-2c}{2(a+b)}$
6.  $3\frac{3}{4}$ , or  $-4$ .
7. 41; 14.

65.

1.  $\frac{4}{3x}$

2.  $x^2(x^2-1)(x^2-4)$ .

3. 3.

4.  $x=5, y=4, z=3$ .

5.  $2x$ .

6.  $b$ , or  $-\frac{ab}{a+b}$ .

66.

1. 520; 480.

2.  $\frac{2x^2+1}{3y^2+y}$

3.  $5x^2+1$ .

4.  $m+1-\frac{1}{m}$

5.  $x=\frac{3a}{2}, y=\frac{3b}{2}$

6.  $\frac{1}{2}$ , or  $-1$ .

67.

1.  $y=\frac{2x}{1+x^2}$

2. 1.

3.  $a^{ps+qr}$ .

4.  $\frac{c^2-ab}{a+b}$

5.  $x=11, y=1$ .

6. 4 miles an hour.

68.

1.  $a-b; -\frac{a^2+ab+b^2}{a}$

2.  $\frac{1}{8}x^3+\frac{11}{8}x^2-\frac{7}{8}x+\frac{1}{8}$ .

3.  $x^2+5x+6$ .

4.  $\frac{x}{x^2+y^2}$

5.  $x=6, y=15$ .

6.  $\frac{a^3}{27}-\frac{ab}{c}+\frac{9b^2}{ac^2}-\frac{27b^3}{a^3c^3}$

69.

1.  $x=a$ , or  $x=b$ , or  $x=c$ , or  $x=d$ .

2.  $(m+n+p+q)(m-n-p+q)$ . 3. 12.

4. A \$5000; B \$5000; C \$10,000. 5.  $a-y$ . 6.  $a+b$ , or 0.

70.

1.  $\frac{1}{14}$ .

2.  $(a^2 - 2bc)^2$ .

3.  $2x^2 - 3x + 4$ .

4. 1.

5.  $\frac{2(x+2)}{(x^2-1)(x^2+x+1)}$ .

6.  $x = \frac{1}{2}$ ,  $y = \frac{1}{3}$ ,  $z = \frac{1}{4}$ .

7.  $3\frac{3}{4}$ , or  $-4$ .

71.

1.  $\frac{37}{8}x^2 - \frac{31}{2}x$ .

2.  $\frac{1}{1-x^4}$ .

3.  $a + b + c$ .

4. 750 English; 375 French.

5.  $\frac{2a+b}{2a-b}$ , or  $\frac{2a-b}{2a+b}$ .

6. 4 miles an hour.

72.

1. 1.

2.  $x + \frac{y}{4x} - \frac{y^2}{32x^2}$ , etc.

4.  $x = \frac{b-c}{b+c-2a}$ ,  $y = \frac{c-a}{b+c-2a}$ ,  $z = \frac{a-b}{b+c-2a}$

5.  $-4$ , or  $-21$ .

3.  $\frac{a^3}{c\sqrt[12]{b}}$ .

6.  $x - 2$ .

73.

1.  $\sqrt[6]{c}$ .

2.  $(x-4)(x-5)(x+11)$ .

3.  $\frac{7}{88}$ .

4. 8, 6.

5.  $x = \pm \sqrt{m^2 + n^2}$ .

6.  $4x^2 + 6ax + a$ .

7.  $-8\frac{1}{2}$ .

74.

1.  $12\frac{4}{5}$  yards.

3. 2.

5. 1.

6.  $\frac{1}{2}$ , or  $-\frac{1}{2}$ .

2. 26; 25.

4.  $x + 1$ .

7.  $\left(\frac{b}{a}\right)^{\frac{2}{3}}$ .

1.  $-\frac{1}{2\frac{1}{2}}$ .

75.

2.  $\left(\frac{1}{a^3}, a^{-3}, \frac{a^4}{a^7}\right), (a^2, \sqrt{a^4}, a^{\frac{1}{2}}, a^3 \times a^{-1}), (a^{\frac{1}{2}}, \sqrt{a}, \sqrt[6]{a^3})$ .

3.  $x^2 + 1 + \frac{1}{x^2}$ .

5.  $a^2b^4$ .

4.  $x = 111, y = 77$ .

6. 15 miles; 2 miles an hour.

7. 4 miles an hour.

76.

1.  $x^{\frac{1}{2}} - x^3 - 4x^{\frac{1}{2}} + 6x - 2x^{\frac{1}{2}}$ .

5.  $24\sqrt{3}$ .

2.  $\frac{65}{a}$ .

4.  $\frac{x}{y}$ .

6.  $a^4b^{\frac{1}{2}}$ .

7.  $16\frac{3}{4}$ , or  $20\frac{3}{4}$ .

77.

1. 4 hours.

4. 36 feet a second; 30 feet a second.

2.  $2a$ , or  $2b$ .

3.  $\frac{a}{2}(-1 \pm \sqrt{5})$ .

5.  $\frac{2}{3}$ , or  $\frac{1}{3}$ .

78.

1.  $a^3 + a - a^{-1} - a^{-3}$ .

4.  $\frac{a^{\frac{1}{2}} + b^{\frac{1}{2}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$ .

5. A \$160; B \$400.

3. 4.

6.  $x = \pm 5, y = \pm 3$ .

79.

1.  $\frac{2}{3}$ .

4. 7; 8.

2.  $(a-1)(a+1)(a-2)(a+2)$ .

5.  $134\frac{1}{2}$  miles.

3.  $\frac{(a-b)^2}{2b}$ .

6.  $\frac{1}{2a}(-b \pm \sqrt{b^2 - 4ac})$ .

$a = c$ .

80.

1. 100 lbs.

2.  $-4480a^3b^{13}$ .

3.  $4m^2 + \frac{2}{3}m + 1$ .

6.  $\frac{b-a}{2b-a}$ , or  $-\frac{b+a}{2b+a}$ .

4.  $x - a$ .

5.  $5xyz^3$ .

7. 5; 2.

## 81.

- |                        |                             |
|------------------------|-----------------------------|
| 1. $c$ .               | 5. $x = 6, y = 5$ .         |
| 2. $x^2 + y^2 + z^2$ . | 6. 12; 13.                  |
| 3. 20.                 | 4. 4.                       |
|                        | 7. $(x+1)(x-2)(x+3)(x+4)$ . |

## 82.

- |  |                                      |
|--|--------------------------------------|
| 1. 4.  | 4. $x$ .                             |
| 2. 10.   | 6. $4x^2\sqrt{xy^3}$ .               |
| 3. $a^6 - b^6$ .   | 7. $x = y = 2$ , or $-\frac{1}{2}$ . |
| 5. $-\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$ . The roots are real, equal, or imaginary, according as $b^2 > 4ac$ , $b^2 = 4ac$ , or $b^2 < 4ac$ . |                                      |

## 83.

- |  |  |
|--|--|
| 1. $x - 1$ .   | 3. $3\frac{1}{2}$ days.                  |
| 2. $\frac{63a}{250c}\sqrt[10]{a^4b^{45}}$ .                                  | 4. $\frac{3}{a}(2 \pm \sqrt{a^2 + 4})$ . |
| 5. 3.  |  |
| 6. $a^6 - 18a^5x + 135a^4x^2 - 540a^3x^3 + 1215a^2x^4 - 1458ax^5 + 729x^6$ . |  |

## 84.

- |  |              |   |
|--|--------------|---|
| 2. $\frac{9x-1}{1-9x^2}$ .   | 3. 16 miles. | 6. $x = \frac{1}{2}, y = \frac{1}{2}$ . |
|  |              | 7. $2a - b$ , or $a + 2b$ .             |
| 4. $(x^4 + \frac{1}{16})(x^2 + \frac{1}{4})(x + \frac{1}{2})(x - \frac{1}{2})$ . |              |   |

## 85.

- |   |   |                           |
|---|---|---------------------------|
| 2. $\frac{2+3x}{1+5x}$ .  | 3. $\frac{40}{b^4}\sqrt[10]{\frac{a^7}{b^6}}$ . | 4. $\pm 3$ , or $\pm 1$ . |
|   |   | 5. $5 - 2\sqrt{2}$ .      |
| 6. $1 - \frac{15y}{4} + \frac{45y^2}{8} - \frac{135y^3}{32} + \frac{405y^4}{256} - \frac{243y^5}{1024}$ . |   |                           |

## 86.

1.  $2a + 6b$ , or  $a - 8b$ .      4.  $16x^4 - 96x^3y + 216x^2y^2$   
 $2. \frac{1+x}{1+x^2}$       3. 2.       $- 216xy^3 + 81y^4$ .  
 5. 7 hours; 5 hours.

## 87.

1. 9.      3. 1.      5.  $32\frac{8}{11}$  minutes past 6.  
 2. 16.      4.  $\frac{c(a-b)}{a}$ .      6. 3 inches.

## 88.

1.  $\frac{12 \times 11 \times 10}{1 \times 2 \times 3} (2x)^9 (-5y)^3 = -14080000 x^9 y^3$ .  
 2.  $\frac{a}{2x^2}$ .      3.  $x = \frac{c}{2a}(a+b)$ ,  $y = \frac{c}{2a}(a-b)$ .  
 4. The carriage, \$300; the horse, \$375; the harness, \$45.  
 5.  $\frac{a}{2} - 3b$ , or  $-(5a+b)$ .

## 89.

1.  $\pm 2$ , or  $\pm 1$ .      3.  $-\frac{1}{8}a^{\frac{1}{2}}b^{-\frac{1}{2}}c^{\frac{1}{2}}$ .  
 2. 0, or  $a$ .      4.  $x=a$ ,  $y=0$ .      5. 300.

## 90.

1.  $\pm 9a^{-\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}}$ .      5. 6 rods.  
 2.  $\sqrt[3]{3}$ .      3.  $a^{\frac{1}{2}}$ .      6.  $x=3$ , or  $3\frac{7}{8}$ ;  $y=2$ , or  $1\frac{1}{8}$ .

## 91.

1.  $\frac{a^3 b^{\frac{1}{2}} x^5 y^{\frac{1}{2}}}{16}$ .      4.  $\frac{1}{a^2}$ , or  $\frac{1}{4a^2}$ .  
 2. Rate, 20 miles; distance, 140 miles.  
 5.  $\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} \times 3^5 \times 4^4 \times x^{\frac{1}{2}} \times y^{\frac{1}{2}}$ .

## 92.

1.  $\frac{7}{2}a$ .

2.  $\frac{-y^3}{2x(x-y)^2(x+y)}$ .

3.  $x - \frac{2}{3}y^{\frac{1}{2}}$ .

4.  $\left(\frac{2ab}{b-2a}\right)^2$ .

5.  $x=2$  or  $\frac{1}{2}$ ,  $y=2$  or  $\frac{1}{2}$ .

6. 5 miles an hour.

## 93.

1.  $(c-a)\sqrt{5m}$ .

2.  $2x(x^2+xy+y^2)$ .

3.  $\frac{2a-b}{ac}$ , or  $-\frac{3a+2b}{bc}$ .

4. 72 feet.

5. The former.

6.  $x=4$ , or 2;  $y=2$ , or 4.

## 94.

1.  $x=\pm 2$ , or  $\pm\sqrt{2}$ ;  $y=\pm 4$ , or  $\pm 3\sqrt{2}$ .

2.  $-\frac{35x}{y}$ ;  $\frac{35y}{x}$ .

3. 5, or  $5\frac{1}{2}$ .

5.  $x^2-13x+42=0$ ; 6, or 7.

6. 84; 48.

## 95.

1.  $m=8$ .

2. 5.

3. 25; 20.

4. 3, or  $\frac{2}{3}(-1 \pm \sqrt{-3})$ , or  $-\sqrt[3]{7}$ .

5.  $\frac{y^{\frac{1}{2}}}{(bx)^{\frac{1}{2}}}$ .

6.  $x-1$ .

## 96.

1.  $\frac{x^3+4y^3+z^3+2xy+xz-2yz}{x^2}$ .

2.  $xy^2$ ;  $918a^3b^3x^5y^4$ .

3.  $x^2 + \frac{1}{x} - \frac{1}{3}$ .

4.  $x=\pm\frac{2}{3}$  or  $\pm\sqrt{\frac{1}{2}}$ ,  $y=\pm\frac{1}{2}$  or  $\mp\sqrt{2}$ .

5. Each root  $= -\frac{b}{a}$  if  $b^2=ac$ .

6.  $1\frac{1}{3}$ .



## 97.

1.  $x^{\frac{1}{2}}y^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}y^{\frac{1}{2}}z + \frac{5}{2}x^{\frac{5}{2}}y^{\frac{1}{2}}z^2 - \frac{5}{4}xy^{\frac{1}{2}}z^3 + \frac{5}{16}x^{\frac{3}{2}}y^{\frac{1}{2}}z^4 - \frac{1}{8}y^{\frac{1}{2}}z^5$ .
2.  $\frac{89 + 28\sqrt{10}}{9}$ ; 19.726.....
3. 4, 1,  $3\frac{1}{2}$ , or  $1\frac{1}{2}$ .
4. Distance, 15 miles; and the rates of walking, 3,  $2\frac{1}{2}$ , and 4 miles.

## 98.

1.  $\frac{1}{8}(\sqrt{2} + \sqrt{3} + \sqrt{5})$ .
2.  $\frac{1}{1-x^2}$ .
3. 1, or  $\frac{1}{21}$ .
4.  $a^2 - b^2$ .
5.  $x=5$ , or  $\frac{5}{4}$ ;  $y=3$ , or  $-1\frac{1}{4}$ .
6.  $16x^2\sqrt[5]{x} - 16x^2y^2\sqrt[5]{x^3} + 6xy^4\sqrt[5]{x^3} - y^6\sqrt[5]{x^4} + \frac{y^8}{16}$ .

## 99.

1. 0.
2.  $1 + 2y^{-\frac{1}{2}} - 3y^{-\frac{3}{2}} + 4y^{-\frac{5}{2}}$ .
3.  $\frac{16x^4}{y^6} - \frac{32x^2}{3y^5} + \frac{8x^3}{3y^3} - \frac{8x^{\frac{1}{2}}y}{27} + \frac{x^3y^4}{81}$ .
4. 12, or 3.
5.  $\frac{a-2b}{2a-3b}$ , or -1.
6. The volume of the 6-inch sphere is equal to the sum of the volumes of the other three spheres.

## 100.

1.  $\frac{2\sqrt{a^2-x^2}}{x^2}$ .
2. 5.414.
3.  $x^2 - 2ax + a^2 + 5b = 0$ .
4. -8, or  $-\frac{1}{8}$ .
5. 3 hours, and 5 hours.

## 101.

1.  $\frac{52}{a^4b^6c}$ .
2. 5, or  $-11\frac{1}{2}$ .
3.  $\frac{1}{4}(6+7\sqrt{3}+3\sqrt{7}+2\sqrt{21})$ .
4.  $x=4$  or  $-\frac{1}{4}$ ,  $y=1$  or  $-3\frac{1}{2}$ .
5. 2.
6.  $-\frac{2}{x-1} - \frac{10}{x-2} + \frac{18}{x-3}$ .
7.  $1 - \frac{5x}{2} + \frac{5x^2}{2} - \frac{5x^3}{4} + \frac{5x^4}{16} - \frac{x^5}{32}$ .

## 102.

1.  $-\frac{5d^3}{3c^{m+2}}$ .
2.  $12 + 3\sqrt[3]{20} - 12\sqrt[3]{3} - \sqrt[3]{180}$ .
3.  $x = 4$ , or  $-1$ ;  $y = 1$ , or  $-4$ .
4. 19, or 18.
5.  $x = 1$ , or  $\frac{1}{3}\sqrt[3]{3}$ .
6.  $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \dots$   
 $+ \frac{1 \times 3 \times 5 \dots (2r-1)}{2 \times 4 \times 6 \dots (2r)} x^r.$

## 103.

1.  $\sqrt[4]{\frac{a+b}{a-b}}$
2.  $x^4 - x^2y^2 + y^4.$
3.  $\pm \frac{7}{6}.$
4. \$2704; \$103.
5.  $-\frac{5}{1024}a^{-4}b^{18}.$
6.  $x = \pm 2$ , or  $\pm \frac{4}{3}\sqrt{3}$ ;  $y = \pm 6$ , or  $\pm \frac{10}{3}\sqrt{3}.$

## 104.

1.  $\sqrt[6]{4000}.$
2.  $\frac{16 + 4\sqrt{2} - 4\sqrt{3} - \sqrt{6}}{13}.$
3. 1, or  $1 \pm 2\sqrt{15}.$
4. 24 inches.
5.  $715 \frac{(2y)^4}{(3x)^{14}}.$
6.  $\frac{1}{6(x+1)} - \frac{1}{2(x-1)} + \frac{4}{3(x-2)}.$

## 105.

1.  $\sqrt[6]{a^2 - b^2}.$
2.  $\frac{am}{n-m}$ , or  $-\frac{am}{n+m}.$
3.  $\pm 5$ , or  $\pm 3\sqrt{2}.$
4.  $x = 7, -1$ , or  $3 \pm \sqrt{-3}.$   
 $y = 1, -7$ , or  $-3 \pm \sqrt{-3}.$
5.  $-1.$
6.  $-1,400,000a^{13}.$

## 106.

1.  $-\frac{21a}{bc^4}.$
2.  $5 + \sqrt{3}.$
3.  $3, -\frac{1}{2}, \frac{1}{2}(5 \pm \sqrt{1329}).$
5. 3, 5, 7.
6.  $1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 - \frac{5}{8}x^4 + \dots$

## 107.

1.  $460 + 11\sqrt{70}$ .
2.  $a^3 b^{\frac{1}{2}} c^{\frac{1}{4}}$ .
3.  $-8, -\frac{1}{8}$ .
4.  $x = \pm 5, y = \pm 2$ .
5. 13, or 7.
7.  $-\frac{10x^4}{243}$ .

## 108.

1.  $6\sqrt{3} - 13$ .
2. 4, 8, 12, 16, 20, 24, .....
3.  $x = 2$ , or  $\frac{8}{10}$ ;  $y = \frac{1}{2}$ , or  $-\frac{8}{9}$ .
4. 60.
5.  $5\frac{2}{5}$ .
6.  $\frac{1}{4a^3(x-a)} - \frac{1}{4a^3(x+a)} - \frac{1}{2a^2(x^2+a^2)}$ .
7.  $-\frac{625x^3}{54}$ .

## 109.

1. 24, 44, 80.
2.  $a$ , or  $\frac{b}{a}(a+b)$ .
3. 16 and 12.
4.  $-\frac{47}{8}$ .
5.  $\frac{5}{7}$ .
6.  $1 + \frac{3}{2}x^2 + \frac{15}{8}x^4 + \frac{35}{16}x^6 + \frac{315}{128}x^8 + \dots$   
 $+ \frac{3 \times 5 \times 7 \dots (2r+1)}{2 \times 4 \times 6 \dots (2r)} x^{2r}$ .

## 110.

1.  $a^{\frac{1}{2}} b^{\frac{1}{4}}$ .
2.  $\frac{1}{8}(14 - 4\sqrt{10})$ .
3. 4, or  $-1$ .
4. \$2650.
5.  $\frac{2205}{4}$ .
6.  $\frac{2n(2n-1) \dots (n+1)}{1 \times 2 \dots n} x^n$ .
7.  $\frac{1}{5(x-7)} - \frac{1}{5(x-2)}$ .

## 111.

1. 1.
2.  $x + 3$ .
3. 10 per cent.
5. 2, 4, 6, 8, 10.

## 112.

1.  $x = \pm 1$ , or  $\pm 4$ ;  $y = \pm 3$ , or  $\pm 2$ .
2. 126 officers, 4000 men.
3. 1, 3, 5, 7, .....
4.  $x = 3, -\frac{11}{2}$ , or  $\frac{-5 \pm \sqrt{33}}{4}$ .
5. 21.

## 113.

1.  $x = \pm 2$ , or  $\pm 3\frac{1}{2}$ ;  $y = \pm 3$ , or  $\pm 2\frac{1}{2}$ .  
 2.  $\frac{22}{15}\sqrt{15}$ . 3.  $x = -3\frac{1}{2}$ . 5. 15 inches.

## 114.

3.  $\frac{2x^2 - 7xy + 5y^2}{x^2 - 5xy + 3y^2}$ . 4.  $\frac{3}{2}$ . 5. 1,556,100.

## 115.

2. 2520. 3.  $x = \frac{1 \pm \sqrt{5}}{2}$ .  
 4. The means are 11, 16, 21, ....; and the sum is 402.  
 5. 200 miles.

## 116.

1. 3, 6, 9, 12, ....  
 2.  $x = \frac{a-b}{2} \pm \frac{1}{2}\sqrt{a^2 + 2ab - 3b^2}$ ,  $y = \frac{a-b}{2} \mp \frac{1}{2}\sqrt{a^2 + 2ab - 3b^2}$ ; or  $x = y = 0$ , or  $a + b$ .  
 5. The scale of 9.

## 117.

3.  $x = 1, -3, 3, -1$ ;  $y = 2, 10, -10, -2$ .  
 4. 2. 5. 0. 6. 16 inches.

## 118.

1.  $2\frac{1}{2}, 4\frac{1}{2}, 6\frac{1}{2}, \dots$  3.  $x^4 + 4x^2 - 96 = 0$ .  
 2.  $3, \frac{3}{4}, \frac{3}{16}, \dots$ ; or  $1, \frac{3}{4}, \frac{9}{16}, \dots$  4. \$1250.  
 5.  $x = \frac{3b^2 \pm \sqrt{3b(4a^3 - b^3)}}{6b}$ ,  $y = \frac{-3b^2 \pm \sqrt{3b(4a^3 - b^3)}}{6b}$ .

## 119.

1.  $\pm 2a$ , or  $\pm a\sqrt{-6}$ . 4.  $y = \frac{16}{x^2} + \frac{4}{x} + 1$ .  
 2. 7, -4,  $\frac{1}{2}(3 \pm \sqrt{349})$ . 5. 6, 54.

## 120.

3. 27.

4. 15, 3.

5. 16.

## 121.

1. 288.

2. 9, 12; or 1, -4.

3.  $b^2 = 5ac$ .

4.  $x = \frac{1}{2} \left( a \pm \sqrt{\frac{4b - a^2(2+m)}{2-m}} \right)$ ,  $y = \frac{1}{2} \left( a \mp \sqrt{\frac{4b - a^2(2+m)}{2-m}} \right)$ .

5. 25 cents.

## 122.

1.  $x = \frac{-ac \pm \sqrt{a^2bc + bd - cd}}{b-c}$ ,  $y = \frac{-ab \pm \sqrt{a^2bc + bd - cd}}{b-c}$ .

If  $b = c$ ,  $x = y = \frac{c}{b}$  or  $\infty$ .

2.  $2a^2$ .

4.  $\frac{28}{125} a^{-\frac{1}{5}} b^3 x^3$ .

3.  $-\frac{9}{2(x-1)} - \frac{20}{x-3} + \frac{61}{2(x-5)}$ .

5. 2520.

## 123.

1. 25.

4.  $x = \pm\sqrt{3}$ , or  $\pm 2$ ;

2. 3, 6, 9, .....

$y = 0$ , or  $\pm 1$ .

3. 18 inches.

5. 30,240.

## 124.

1. 100.

2. 2, 8, 32, 128; or  $16\frac{1}{2}$ ,  $28\frac{1}{2}$ ,  $46\frac{1}{2}$ ,  $78\frac{1}{2}$ .

3.  $5, 3 \pm \sqrt{2}$ .

5. 814.

4.  $x = 3, -\frac{2}{3}, y = 2, \frac{1}{3}$ .

6. 10 and  $10\frac{1}{2}$  miles per hour.

## 125.

1. 25. 2.  $\sqrt[3]{4}, \sqrt[3]{2}, 1, \sqrt[3]{\frac{1}{2}}, \sqrt[3]{\frac{1}{4}}$ . 4. Equal amounts.

5.  $-\frac{m(m+1) \cdots (m+8)}{1 \times 2 \cdots 9} \times x^{-(m+9)} \times y^9$ .

## 126.

2.  $\pm \frac{a(m+n)}{\sqrt{2m^2+2n^2}}$ . 5. 1, 3, 9, 15; or 16, 8, 4, 0.  
 3.  $x = \pm 10$ ,  $y = \pm 5$ . 6.  $123,410x^{39}$ .  
 4.  $\frac{4}{3}, \frac{2}{3}, \frac{2}{3}, \frac{1}{3}, 0, -\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}$ .

## 127.

1.  $9 + \frac{11}{8}\sqrt{6}$ . 3. 12, or 8.  
 2.  $x = 8$ , or 4;  $y = 4$ , or 8. 4. 5, 7, 9, 11, 13. 5. 16.  
 6.  $x = -\frac{1}{2}(y-1) + \frac{1}{8}(y-1)^2 - \frac{9}{16}(y-1)^3$   
 $+ \frac{1}{128}(y-1)^4 - \dots$

## 128.

1.  $6 - 6\sqrt[3]{6}$ . 4. 6, 8, 10. 5. 1536.  
 2. 1, or  $\frac{1}{2}\sqrt[3]{3}$ . 6.  $10 \times 5^9 \times x^9$ .  
 3. 240 feet, very nearly. 7. Septenary.

## 129.

1.  $a, \sqrt{a^2-b^2}, \frac{a^2-b^2}{a}$ . 2. 24.  
 3.  $x = 5, -3$ , or  $1 \pm \frac{1}{2}\sqrt{-88}$ ;  $y = 3, -5$ , or  $-1 \pm \frac{1}{2}\sqrt{-88}$ .  
 4.  $l = -\frac{d}{2} \pm \sqrt{\left(\frac{d}{2} - a\right)^2 + 2ds}$ .  
 5.  $\frac{a^4}{81} + \frac{8a^5x}{243} + \frac{40a^6x^2}{729} + \frac{160a^7x^3}{2187} + \frac{560a^8x^4}{6561}$ .  
 6.  $\frac{6}{7(x-5)} - \frac{1}{5(x-3)} + \frac{12}{35(x+2)}$ . 7.  $[6 \times [3]$ .

## 130.

1.  $\frac{1}{16}$ . 4. 5 days. 5. 12.  
 2.  $\pm 5$ , or  $\pm 4\sqrt{2}$ . 6.  $-2, \infty, 2, 1, \frac{1}{2}$ .  
 3. 8 feet; 4 feet. 7.  $6a^{-14}x^{10}$ .

## 131.

1.  $\frac{1}{2}(3 \pm \sqrt{5})$ , or  $\frac{1}{2}(9 \pm \sqrt{-89})$ .
2.  $x = \pm 2$ , or  $\pm \frac{4}{3}\sqrt{3}$ ;  $y = \pm 6$ , or  $\pm \frac{10}{3}\sqrt{3}$ .
3. 8.                      5.  $1 + 3x + 4x^2 + 7x^3 + 11x^4 + 18x^5 + \dots$
4. 10, 12, 15.    6. 84.

## 132.

1.  $\frac{1}{2}(3 + 2\sqrt[3]{20} - 2\sqrt[3]{50})$ .    5. 16.
2. 16, or  $(-\frac{11}{8})^4$ .                      6.  $\lfloor 7 \times \rfloor 4$ .
3.  $x = (a \pm b)$ ,  $y = (a \mp b)$ .
4. 3,  $\frac{10}{8}$ ,  $\frac{11}{8}$ , .....; and 5,  $\frac{16}{8}$ ,  $\frac{17}{8}$  .....

## 133.

1.  $x = \frac{5}{2}$ , or  $-\frac{3}{2}$ ;  $y = 3$ , or  $-5$ .
2. 78 miles.                      3.  $\frac{3n(n+3)}{8}$ .                      4.  $\frac{8}{21}$ .
5.  $\frac{3}{2(x-6)} - \frac{3}{5(x-3)} + \frac{1}{10(x+2)}$ .
6.  $\frac{\lfloor 12 \rfloor}{\lfloor 4 \rfloor \lfloor 8 \rfloor} \times \frac{\lfloor 8 \rfloor}{\lfloor 3 \rfloor \lfloor 5 \rfloor}$ ;  $\frac{\lfloor 12 \rfloor}{\lfloor 4 \rfloor \lfloor 8 \rfloor} \times \frac{\lfloor 7 \rfloor}{\lfloor 2 \rfloor \lfloor 5 \rfloor}$ .

## 134.

1. 4.                                      4. 2, 5, 8, 11.
2. 5, or  $-30$ .                      5. 2, 8, 32; or  $-2, -8, -32$ .
3.  $x = 9, 3, -6, -9$ ;  $y = 3, 9, -9, -6$ .
6.  $-\frac{7 \times 3^5}{2 \times 4^3} \times \frac{y^{\frac{1}{2}}}{a^3 c^{\frac{1}{2}} x^{\frac{1}{2}}}$ .

## 135.

1.  $\sqrt[36]{524288}$ .    2.  $\left(\frac{a \pm \sqrt{a^2 + 4b}}{2}\right)^{\frac{2n}{p}}$ .    3.  $\frac{3}{2}\sqrt{\frac{a}{g}}$ .    4. 49, 1.
5.  $x = y - 1 + 2(y-1)^2 + 7(y-1)^3 + 30(y-1)^4 + \dots$
6. 6  $\lfloor 4 \rfloor \lfloor 4 \rfloor$ .

## 136.

1.  $x = \pm \sqrt{\frac{a}{2b}}$ ,  $y = \pm \sqrt{\frac{b}{2a}}$     5.  $\frac{\overline{36}}{\overline{5} \overline{31}}; \frac{\overline{35}}{\overline{4} \overline{31}}$ .  
 2. 64.  
 3. 16, 24, 36, .....    6.  $x = y + \frac{y^2}{2} + \frac{y^3}{3} + \dots$

## 137.

1. 1,  $-\frac{1}{2} \pm \frac{1}{2}\sqrt{-3}$ .    3. 33.    5. 44,330.  
 2.  $x = 5$ , or  $-3\frac{1}{2}$ ;  $y = 20$ , or  $-15$ .    4. 194.    6. 123,225.

## 138.

1.  $x = 0$ , or  $\frac{1}{2b}[a + 2 \pm \sqrt{a^2 - 4}]$ ;  
 $y = 0$ , or  $\frac{1}{2b}[a + 2 \mp \sqrt{a^2 - 4}]$ .  
 2. At 7 hours 48 minntes, the first time; and then at intervals of 2 hours 24 minutes.  
 3.  $\frac{1}{2}\frac{2}{5}$ .    4. 44,100.  
 5.  $x = y - y^2 + y^3 - 4y^5 + 14y^8 - 30y^7 + \dots$

## 139.

1.  $x = \frac{1}{2}a[\pm\sqrt{-1} \pm \sqrt{3}]$ ,  $y = \frac{1}{2}a[\pm\sqrt{-1} \mp \sqrt{3}]$ .  
 2. 10;  $\frac{\overline{20}}{\overline{10} \overline{10}}$ .    4. 3, 2, 1, 4; or 1, 3, 3, 3.  
 3.  $\frac{2}{3}\frac{2}{3}\frac{2}{3}$ .    5.  $x = \pm 1$ , or  $\frac{3 \pm \sqrt{5}}{2}$ .  
 6.  $\frac{7}{11}\frac{7}{11}$ .

## 140.

1. They are reciprocals.  
 2.  $x = \pm 2$ , or  $\pm \sqrt{-1}$ ;  $y = \mp 1$ , or  $\mp 2\sqrt{-1}$ .  
 3. 2.00498.    5.  $2 - x - x^2 + 2x^3 - \dots$   
 4.  $\frac{1+3x}{(1-x)^2}$ .    6.  $x = 9$ ,  $y = 2$ .



## 141.

- |  |            |           |
|--|------------|-----------|
| 2. 3, -6, $\frac{4}{3}$ .  | 4. 146.    | 5. 15.68. |
| 3. $\log a + \frac{m}{n} \log c + \operatorname{colog} b + \frac{1}{2} \operatorname{colog} d$ . | 6. 1.0788. |           |

## 142.

- |                                |                  |             |
|--------------------------------|------------------|-------------|
| 2. 2, -4, 12, $-\frac{5}{7}$ . | 4. 5.81238 - 10. | 6. 0.01987. |
| 3. 9.00000 - 10.               | 5. 0.02524.      |             |

## 143.

- |                            |             |             |           |
|----------------------------|-------------|-------------|-----------|
| 2. -3, -4, $\frac{1}{2}$ . | 3. 0.87174. | 4. 0.71426. | 5. 1.537. |
|----------------------------|-------------|-------------|-----------|

## 144.

- |  |           |              |
|--|-----------|--------------|
| 1. $\frac{1}{3}, \frac{2}{3}, \frac{5}{8}, -\frac{4}{3}$ . | 3. 31.1.  | 5. 2.57.     |
| 2. $\frac{1}{4}$ .   | 4. 39.99. | 6. 10 years. |

## 145.

- |  |               |
|--|---------------|
| 1. 3, 2, -4.                                 | 4. 0.04.      |
| 2. $\frac{1}{1000000}, \frac{1}{243}, 243$ . | 5. 0.0001129. |
| 3. $\frac{3}{10}$ .                          | 6. 5.593.     |

## 146.

- |              |            |              |
|--------------|------------|--------------|
| 1. 2, 4, -3. | 3. 0.1941. | 5. 0.585.    |
| 2. 0.5646.   | 4. 1.703.  | 6. \$613.70. |

## 147.

- |                    |               |                |
|--------------------|---------------|----------------|
| 1. 6.              | 3. 0.0001404. | 5. 4.429.      |
| 2. $\frac{1}{3}$ . | 4. 0.2929.    | 6. 36.7 years. |

## 148.

- |            |   |
|------------|---|
| 1. -4.     | 3. 2.94444, 3.13549, 3.36730, 3.43399.    |
| 2. 0.4266. | 4. 8.058.    5. \$29,979.30.    6. 9.588. |

## 149.

2. 1, -1, 3,  $\frac{7}{5}$ ,  $m-n$ ,  $-\frac{n}{m}$ .      3. 1.816.  
4. 1.067.      5. 0.69897, 1.04139.

## 150.

1. 1, 5, -1, 0,  $n$ ,  $\frac{m-n}{m}$ .      4. 1.18329.  
2. 139.8.      5. 1.60529.  
3. 0.2372.      6.  $66\frac{5}{7}$  years.

Press of  
Berwick & Smith,  
Boston.

•

# Mathematics.<sup>2</sup>

		<i>Introd. Prices.</i>
Byerly.....	Differential Calculus.....	\$2.00
	Integral Calculus.....	2.00
Ginn.....	Addition Manual.....	.15
Halsted.....	Mensuration.....	1.00
Hardy.....	Quaternions.....	2.00
Hill.....	Geometry for Beginners.....	1.00
Sprague.....	Rapid Addition.....	.10
Taylor.....	Elements of the Calculus.....	1.80
Wentworth.....	Grammar School Arithmetic.....	.75
	Shorter Course in Algebra.....	1.00
	Elements of Algebra.....	1.12
	Complete Algebra.....	1.40
	Plane Geometry.....	.75
	Plane and Solid Geometry.....	1.25
	Plane and Solid Geometry, and Trigonometry.....	1.40
	Plane Trigonometry and Tables. <i>Paper</i> ..	.60
	Pl. and Sph. Trig., Surv., and Navigation.....	1.12
	Pl. and Sph. Trig., Surv., and Tables.....	1.25
	Trigonometric Formulas.....	1.00
Wentworth & Hill:	Practical Arithmetic.....	1.00
	Abridged Practical Arithmetic.....	.75
	Exercises in Arithmetic.....	
	Part I. <i>Exercise Manual</i> .....	
	Part II. <i>Examination Manual</i> .....	.35
	Answers (to both Parts).....	.25
	Exercises in Algebra.....	.70
	Part I. <i>Exercise Manual</i> .....	.35
	Part II. <i>Examination Manual</i> .....	.35
	Answers (to both Parts).....	.25
	Exercises in Geometry.....	.70
	Five-place Log. and Trig. Tables ( <i>7 Tables</i> ).....	.50
	Five-place Log. and Trig. Tables ( <i>Comp. Ed.</i> ).....	1.00
Wentworth & Reed:	First Steps in Number, <i>Pupils' Edition</i> .....	.30
	<i>Teachers' Edition</i> , complete.....	.90
	Parts I., II., and III. (separate), each.....	.30
Wheeler.....	Plane and Spherical Trig. and Tables.....	1.00

*Copies sent to Teachers for examination, with a view to Introduction,  
on receipt of Introduction Price.*

**GINN & COMPANY, Publishers.**

BOSTON.

NEW YORK.

CHICAGO.

# GINN & COMPANY'S BOOKS.<sup>1</sup>

---

**ELEMENTARY ENGLISH :** Classics for Children: Water-Babies, Robinson Crusoe, Swiss Family Robinson, Greek Heroes, Tales from Shakespeare, Lady of the Lake, Tales of a Grandfather, Quentin Durward, Sketch Book, Merchant of Venice, Stories of the Old World; Stickney's Primer; Turner's Primer and First Reader, Stories; Whitney & Knox Language Series: Elementary Lessons (*Pupils' edit.*, *Teachers' edit.*), Grammar; Hazen's Speller.

**HIGHER ENGLISH:** Arnold's Literature; Bancroft's Composition; Fulton & Trueblood's Choice Readings; Hudson's Harvard and School Shakespeares; Sprague's Irving, Milton.

**ANGLO-SAXON:** Carpenter's Grammar, English of XIV. Century; Sievers' Grammar; Beowulf; Exodus & Daniel; Beowulf (Trans.).

**LATIN:** Allen & Greenough's Grammar, Cæsar, Sallust, Cicero, Ovid; Allen's New Method, Composition, Tacitus; Blackburn's Grammar and Exercises; Greenough's Virgil; Leighton's Lessons; Shumway's Synonyms; Tetlow's Lessons; White's Lexicons; Whiton's Six Weeks' Preparation; Classical Atlas.

**GREEK:** College Series of Greek Authors; Flagg's Demosthenes, Seven against Thebes; Goodwin's Grammar, Reader, Moods and Tenses, Xenophon and Herodotus; Goodwin & White's Anabasis; Leighton's Lessons; Liddell & Scott's Lexicons; White's Lessons, Oedipus Tyrannus; Lysias; Pindar; Lyric Poets.

**MATHEMATICS:** Byerly's Calculus; Hill's Geometry for Beginners; Taylor's Calculus; Wentworth's Arithmetic, Algebras, Geometries, Trigonometries, Surveying, Navigation; Wentworth & Hill's Practical Arithmetic, Exercises in Arithmetic, Algebra, and Geometry; Wentworth & Reed's First Steps in Number (*Pupils' edit.*, *Teachers' edit.*); Wheeler, Trig. and Tables.

**MUSIC:** Mason's First, Second, Third, Intermediate, and Independent Readers, Charts, Hymn and Tune Books; Sharland's Fourth Readers; Eichberg's High School Readers.

**GEOGRAPHIES, GLOBES, AND MAPS:** Fitz's Globes; Ginn & Heath's Classical Atlas; Hall's Our World, No. 1 and No. 2; Johnston's Wall Maps; Joslin's Globes.

**SCIENCE:** Gage's Physics.

**PHILOSOPHY:** Ladd's Lotze's Outlines; Seelye's Hickok's Moral and Mental Science.

**MODERN LANGUAGES:** Knapp's Spanish Grammar and Readings, French Readings.

**SANSKRIT:** Perry's Primer; Lanman's Reader; Whitney's Grammar.

AND MANY OTHERS.

---

FULL DESCRIPTIVE CATALOGUE FREE.

---

GINN & COMPANY, Publishers, Boston, New York, and Chicago.

